Can An Algebraic Diagnostic Test be Used to Predict Final Grades in an Introductory Statistics Class?

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ABSTRACT

The Department of Mathematics and Statistics at California State University Sacramento has been using the Intermediate Algebra Diagnostic (IAD) test as a proxy tool to screen students intending to enroll in an introductory statistics course (Stat 1). However, the use of an algebraic test as a diagnostic tool for a statistics course has been questioned by some faculty members and students at this university. The regression models used in this study (simple linear regression, hierarchical linear regression, and logistic regression) show that higher IAD scores are related to higher final grades in Stat 1, even after adjusting for different instructors. Inferences were also made in this study to predict a passing grade and passing rates in Stat 1 based on the bounds of the confidence and prediction intervals obtained for the IAD scores with these models.

KEYWORDS

Linear Regression, Logistic Regression, Statistics Diagnostic Test, Algebra Diagnostic Test, MDTP, Introductory Statistics

1. BACKGROUND

Past experience among faculty members in the Department of Mathematics and Statistics at California State University, Sacramento (CSUS) indicates the need to require some level of abstract reasoning and quantitative maturity from students registering in the course Introduction to Statistics (Stat 1). The department has never been able to find an appropriate existing diagnostic test for introductory statistics courses, so it has been using the Intermediate Algebra Diagnostic (IAD) test as a proxy tool to screen students intending to enroll in Stat 1. However, the use of an algebraic test as a diagnostic tool for a statistics course has been questioned by some faculty members and students at CSUS. Therefore there is a need for a study to determine if there is a relationship between the IAD scores and final grades in Stat 1. For this purpose the IAD scores and final grades in Stat 1 of over 550 students who have taken this course over a period of two semester terms and one summer session were collected to conduct such a study.

The IAD test is a product of the Mathematics Diagnostic Testing Project (MDTP)[1]. This project was founded in 1978 as a product of a joint endeavor between the University of California and the California State University systems, which aimed to design a measurement of student readiness for a wide range of mathematics courses. The original test was created not only by faculty members from these two university systems, but also by a workgroup of faculty from community colleges and high schools in California[1]. The current workgroup of developers of the test is assisted by the Educational Testing Service[1].

The IAD test and other tests created by the MDTP do not provide a scaling score (such as A, B, C, etc.) since these tests are not intended to be used as final exams. Instead, raw scores (0 to 45 in the case of the IAD test) are reported to students to complement a placement procedure that should take into account previous coursework, grades, etc., as part of a reliable indicator of students'

mathematical proficiency to succeed in a course[1]. Currently, the Mathematics and Statistics Department at CSUS requires a minimum IAD score of 32 for students to register in Stat 1 classes; however students with scores from 27 – 31 can sometimes register with a warning that they may have to work harder than the rest of the class to succeed in the course (the department calls this range of scores "Advisory Enrollment"). These are the same minimum scores required for Modern Business Math (Math 24), Calculus for Social and Life Sciences (Math 26A), and Precalculus (Math 29).

The literature on the Web page of the MDTP project states that in order to determine cutoff scores and "counseling ranges,"[1] statistical analyses of test scores and subsequent course performance should be conducted to validate the use of the test as a placement tool[1]. To date, no record of a statistical analysis of this kind exists for the Stat 1 course. However, the number of full time statisticians in the Mathematics and Statistics department at CSU Sacramento has increased from two to five in recent years, which has provided the department with the manpower needed to pay closer attention to this course, and to conduct such an analysis. The analysis presented here was conducted to investigate the question in the title of this article.

The Science Transfer Project (STP), which is part of the CSUS Science Educational Equity Program (SEE), aims to offer basic research experience to the underrepresented students that it serves. This project seeks to help students at the undergraduate level to learn, with faculty supervision, skills for carrying out elementary research in their fields of specialization. The analysis required for this study provides an opportunity that meets the goals of this program. The didactic style of this article stems from the need for the students involved in this project to both learn linear and logistic regression modeling while also learning how to analyze their results to test research hypotheses.

2. METHODS

The data collected for this analysis contains the IAD scores, final grades, and coded information of the Stat 1 classes for 564 students who took this course during the Summer and Fall of 2008, and the Spring of 2009. The original data set consisted of a larger number of students, but this number was reduced due to the inclusion criteria for this study, which excluded students who were taking this class for at least a second time and students who had taken more advanced mathematics classes prior to taking Stat 1.

Descriptive statistics of the data set are presented in **Table 1** where a general increasing trend in final grades can be observed as the values of the measures of location statistics (mean, median and quartiles) for the IAD scores increase.

Final Grade	n (%)	Min	Q_1	Median	Q_3	Max	$\bar{x}$	S
A	97 (17)	19	31	35	40	45	35.19	5.45
A-	29 (05)	28	31	33	37	41	33.69	3.70
B+	93 (16)	22	30	32.5	35	41	32.52	4.00
B	43 (08)	20	30	33	36	42	32.92	4.90
B-	46 (08)	23	28	31	34	44	31.09	4.38
C+	86 (15)	26	28	31.5	34	42	31.64	4.21
C	38 (07)	18	28	31	34	41	31.03	3.93
C-	36 (07)	24	27	30.5	34	39	31.11	4.17
D+	36 (07)	23	31	32	34	36	31.57	4.20
D	8 (01)	19	28	30	34	39	30.39	4.40
D-	7 (01)	24	29.25	30.5	32.5	37	30.63	4.00
F	45 (08)	19	27	30	33	44	30.47	4.77

Table 1. Summary Statistics for the IAD scores by final grade: n(%) = number (percentage) of students who obtained the grade in column 1; Min = minimum IAD, Max = maximum IAD, Q_1 = 25th percentile, Median, Q_3 = 75th percentile, mean $\bar{x}$, and standard deviation S of the IAD scores of students who obtained the grade in column 1.

The relationship between IAD scores and final grades was analyzed using linear and logistic regression techniques. The linear regression analysis allowed for inferences to be made regarding the minimum IAD scores required to pass STAT 1, while the logistic regression analysis allowed for inferences regarding probabilities of passing STAT 1 for each of the IAD scores observed.

For the linear regression analysis, final letter grades were converted to percentages in order to obtain a numerical response variable. Since each letter grade corresponds to an interval of percentages, two scales were obtained to reflect on the different criteria that can be used when assigning percentages to letter grades. Two regression models using each of these scales were fitted to the data, and inferences using confidence and prediction intervals from these models were subsequently compared. In addition, a hierarchical linear regression model was included to examine the hierarchical nature of the data (students nested within classrooms). For the logistic regression analysis, the grades were converted to a dichotomous outcome, pass or fail, using two criteria. In the first case, passing was defined as obtaining a grade of at least a D-, the general cutoff for a passing grade, while in the other passing was defined as obtaining a grade of at least C-, which is the criterion used by many majors on campus that require Stat 1.

The linear and logistic regression analyses were conducted using the statistical software R[2], and the hierarchical linear regression model obtained using HLM 7[3].

3. ANALYSIS

3.1. Linear Regression Analysis

The linear regression analysis is divided into three sections. In the first section two regression models are fitted using the two scales of percentages assigned to the final letter grades. In the second section these models are used to obtain confidence and prediction intervals for final percentage grades based on IAD scores. The third section presents an analysis using a hierarchical linear regression model to examine the hierarchical nature of the data where students are nested within classrooms.

3.1.1. Linear Regression Models

When using the first scale, the numbers assigned to the final letter grades are the midpoints of the numerical intervals that are used to assign these letters based on the standard scale used at CSUS. For example, students who received an A as a final grade had an average final course percentage that fell into the range of 93 to 100; therefore, these students were assigned a numerical grade of 96.5. For a grade of F, which corresponds to a percentage from 0 to 59.9, a value of 57.95 was assigned to match the difference of 3.5 percentage points that exists between the midpoints for other letter grades. This numerical recoding of the final grades is listed in **Table 2**.

Letter Grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Numerical Grade	96.5	91.45	88.45	84.95	81.45	78.45	74.95	71.45	68.45	64.95	61.45	57.95

Table 2. Numerical coding used for final letter grades (first scale).

In the scatterplot (shown with its least squares regression line) of numerical final scores against IAD scores in **Figure 1**, a positive linear trend is not very obvious to the naked eye due to the overlapping data points which appear in different shades. However, **Figure 2** shows another graphical representation of this data set using boxplots of the numerical scores, without outliers, against each of the values of the IAD scores. In this plot it can be seen that for those students (a total of 41) who were admitted to a Stat 1 class by exception, with an IAD score below the minimum of 27 (see "Advisory Enrollment" in "Background"), the relationship of the IAD scores to the final numerical grades does not follow a linear trend. However, the median numerical final grades for each IAD score (represented by the segment in the middle of the boxplots) follows a general positive linear trend starting at about an IAD score of 26 and 27. Thus a linear regression model was fitted to the entire data set, and then separate linear regression models were fitted for each of the subsets of students with IAD scores below 27 and students with IAD scores of 27 and above. That is, the intercept β_0 and the slope β_1 of the linear equation of these data's numerical final scores, y_i, in terms of the IAD scores, x_i, are estimated via ordinary least squares regression:

$$y_i = \beta_0 + \beta_1 x_i + e_i \qquad \textbf{Equation 1.}$$

The e_i in **Equation 1** represent the effect of the particular set of students i in this study who received a specific IAD score x_i. This model assumes e_i to be normally distributed with mean zero and common variance for the numerical grades across groups of IAD scores.

It is helpful to scale the variable x by subtracting the average of all IAD scores, $\bar{x}$, from the IAD score of each student, $(x_i - \bar{x})$ (a process called "centering" in statistics) so that the intercept in **Equation 1** matches the average of the numerical final grades for all students (the estimate of the slope does not change when centering the variable x):

$$y_i = \beta_0 + \beta_1 (x_i - \bar{x}) + e_i \qquad \textbf{Equation 2.}$$
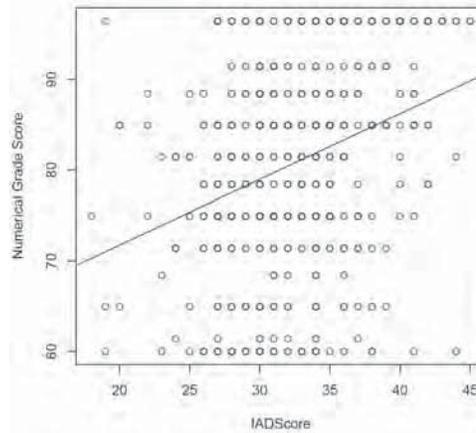
Figure 1. Numerical grade scores against IAD scores with least squares regression line (Scenario 1).
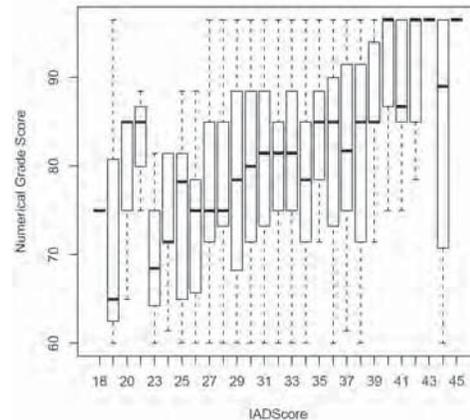

Figure 2. Boxplots of Numerical grade scores against IAD scores (first scale).

The estimated coefficients in **Equation 2** when using the entire data set is:

$$y_i = 80.4727 + 0.7365(x_i - 32.2814) + e_i \qquad \textbf{Equation 3.}$$

Table 3 presents the summary of the estimates in **Equation 3** where it can be seen that there is a significant statistical linear relationship (t-test, 562 d.f., p-value: 5.29 x 10^{-14}) between the IAD scores and the numerical version of the final grades. The slope in **Equation 3** indicates that for each extra unit achieved in the IAD score, the estimated numerical final grade is expected to increase, on average, by about 0.74 of a percentage point. Of less importance, the p-value of the intercept of this centered equation is the p-value for the t-test of the hypothesis that the average numerical final grades for the students in this data set, 80.4727, is different from zero (however, it would be regrettable if this hypothesis were not rejected!).

	Degrees of Freedom	Estimate	Standard Error	t–value	Pr (> | t |)
Intercept	1	80.4727	0.4616	174.34	< 0.001
Slope	562	0.7365	0.0964	7.722	< 0.001
Residual Standard Error: 10.96 R-squared = 0.09409					

Table 3. Summary for the regression model using the entire data set (first scale).

The estimates in **Equation 3** changed only slightly when excluding the 41 students with IAD scores of 26 and below, and also when using students with IAD scores of 27 and above. Therefore, there was no need to provide separate analyses for these subgroups.

The second scale uses a conservative approach when assigning the numbers to the final letter grades: instead of using the midpoints of the numerical intervals, mentioned above, that correspond to these letters, it uses their lower bounds, with the exception of using 50 for the interval [0–59] that corresponds to the letter F. This numerical coding of the letter grades is shown in **Table 4**.

Letter Grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Numerical Grade	93	90	87	83	80	77	73	70	67	63	60	50

Table 4. Numerical coding used for final letter grades (second scale).

The estimates and summary of the regression equation when using the numerical final grades in **Table 4** are shown in **Equation 4** and **Table 5**, respectively. It is no surprise that the slope of **Equation 4** is almost the same as that of **Equation 3** since changing the numerical grades from the first to the second scale amounts to an almost vertical downward shift (numerical grades were decreased by almost the same constant amount) in the data points. The intercept in **Equation 4** does reflect this downward shift in predicted final grades sought with this model.

$$y_i = 77.9592 + 0.7371(x_i - 32.9446) + e_i \qquad \textbf{Equation 4.}$$

	Degrees of Freedom	Estimate	Standard Error	t–value	Pr (> | t |)
Intercept	1	77.9592	0.4917	158.548	< 0.001
Slope	562	0.7371	0.1027	7.177	< 0.001
Residual Standard Error: 11.68 R-squared = 0.08397					

Table 5. Summary for the regression equation using students with IAD scores of 27 and above (second scale).

3.1.2. Confidence Intervals (CI) and Prediction Intervals (PI)

The least squares regression lines provide estimates for the population average numerical final grade for each of the IAD scores. Upper and lower bounds for 95% confidence intervals for the *average* numerical final grade of students broken down by IAD score are listed in **Tables 6** and **7** for models (3) and (4), respectively. These bounds were computed using the formula in **Equation 5** [4]. Assuming the grades and scores of the students included in this study are a representative sample of those of the population of students who take Stat 1, one can be 95% confident that the *average* numerical final numerical grade in Stat 1 of the population of students with a given IAD score falls between these two bounds. Note that, under both models, one can be 95% confident that even students with an IAD score of 18 will get, on average, a final grade of at least 64.3953%, which corresponds to at least a grade of D. On the other hand, students with an IAD score of at least 22 under model (3), and at least 25 under model (4), can be 95% confident to obtain, on average, a final numerical grade above the minimum numerical final grade of 70% (corresponding to a grade of C-) required as a passing grade by most majors on campus that have this class as a requirement.

$$\bar{y} + \hat{\beta}_1 (x_0 - \bar{x}) \pm t_{n-2,1-\frac{\alpha}{2}} S_{y|x} \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{(n-1)s_{\bar{x}}^2}} \qquad \textbf{Equation 5.}$$

IAD	CI:Lower	CI:Upper	PI:Lower	PI:Upper	IAD	CI:Lower	CI:Upper	PI:Lower	PI:Upper
18	67.10279	72.80611	48.23899	91.6699	32	79.35742	81.17348	58.71891	100*
19	68.01811	73.36379	48.99825	92.38365	33	80.08533	81.91857	59.45504	100*
20	68.93188	73.92301	49.75588	93.09902	34	80.77536	82.70154	60.18951	100*
21	69.84375	74.48414	50.51187	93.81602	35	81.43256	83.51734	60.92232	100*
22	70.75325	75.04764	51.26623	94.53467	36	82.06371	84.35919	61.65347	100*
23	71.65976	75.61414	52.01894	95.25496	37	82.67527	85.22063	62.38296	100*
24	72.56243	76.18447	52.77	95.97689	38	83.27243	86.09647	63.11079	100*
25	73.4601	76.75979	53.51942	96.70048	39	83.85903	86.98286	63.83696	100*
26	74.35117	77.34173	54.26718	97.42572	40	84.43784	87.87705	64.56148	100*
27	75.23334	77.93255	55.01328	98.15261	41	85.01082	88.77708	65.28434	100*
28	76.10344	78.53546	55.75773	98.88117	42	85.57935	89.68155	66.00555	100*
29	76.95703	79.15486	56.50052	99.61138	43	86.14444	90.58945	66.72512	100*
30	77.78834	79.79656	57.24164	100.000*	44	86.70684	91.50006	67.44305	100*
31	78.59058	80.46732	57.98111	100.000*	45	87.26709	92.41281	68.15934	100*

Table 6. Confidence Intervals (CI) and Prediction Intervals (PI) for IAD scores (first scale).
*Indicates truncated bounds due to overshoot.

Of more interest for this study is the construction of *prediction* intervals, which differ in a fundamental way from confidence intervals: prediction intervals provide an interval estimate for a future individual numerical final grade of a student who enrolls in Stat 1 with a particular IAD score, rather than the average numerical final grade for a population of individuals who have enrolled in a Stat 1 course and who share a particular common IAD score. The lower and upper bounds for this kind of interval, broken down by IAD score, were obtained using **Equation 6** [4] for each of the models. The bounds of these intervals are also listed in **Tables 6** and **7**.

$$\bar{y} + \hat{\beta}_1 (x_0 - \bar{x}) \pm t_{n-2,1-\frac{\alpha}{2}} S_{y|x} \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{(n-1)s_{\bar{x}}^2}} \qquad \textbf{Equation 6.}$$

IAD	CI:Lower	CI:Upper	PI:Lower	PI:Upper	IAD	CI:Lower	CI:Upper	PI:Lower	PI:Upper
18	64.3953	70.4715	44.29639	90.57041	32	76.78527	78.72019	54.79566	100*
19	65.32292	71.01807	45.05773	91.28326	33	77.51316	79.46649	55.53237	100*
20	66.24889	71.56629	45.81734	91.99784	34	78.20069	80.25314	56.26730	100*
21	67.17283	72.11654	46.5752	92.71417	35	78.85326	81.07477	57.00046	100*
22	68.09425	72.66931	47.33132	93.43224	36	79.47808	81.92414	57.73185	100*
23	69.01248	73.22527	48.08568	94.15207	37	80.08203	82.79438	58.46147	100*
24	69.92661	73.78533	48.83829	94.87365	38	80.67065	83.67995	59.18932	100*
25	70.83543	74.35070	49.58915	95.59698	39	81.24802	84.57676	59.91541	100*
26	71.73719	74.92313	50.33824	96.32208	40	81.81709	85.48188	60.63974	100*
27	72.62949	75.50502	51.08556	97.04895	41	82.37995	86.39321	61.36230	100*
28	73.50890	76.09980	51.83112	97.77758	42	82.93808	87.30928	62.08311	100*
29	74.37073	76.71216	52.57491	98.50798	43	83.49254	88.229	62.80216	100*
30	75.20881	77.34827	53.31693	99.24014	44	84.04413	89.1516	63.51947	100*
31	76.01590	78.01537	54.05718	99.97409	45	84.59344	90.07649	64.23503	100*

Table 7. Confidence Intervals (CI) and Prediction Intervals (PI) for IAD scores (second scale).
* Indicates truncated bounds due to overshoot.

Note that these intervals, on the one hand, indicate that for a randomly chosen student with a grade of at least 34 under model (3), and a at least 40 under model (4), one can be 95% confident that he/she will pass Stat 1 with a numerical final grade of at least 60%, which corresponds to a grade of at least a D-. On the other hand, one could argue that this model also predicts that if this student has an IAD score as low as 18, his/her predicted final grade may be as high an A- (91.6%) — the same can be said with for the model that uses the more conservative scale, although with a lower percentage final grade that still corresponds to an A- (90.57%). Note also that the prediction intervals of IAD scores as low as 30 of this model have upper bounds that needed to be truncated due to overshoot (upper bounds above the maximum of 100%).

It can be then concluded then, that the variation in final grades for each IAD score did not allow the regression models to provide prediction or confidence intervals tight enough to predict accurately, final grades in terms of IAD scores.

3.1.3. Hierarchical Linear and Logistic Regression Models (Adjusting the Model for Classrooms)

The data set of this study consists of students from 32 classes taught by a total of 11 instructors. The linear regression models above assume that the effect of IAD scores is the same across classrooms and instructors. However, it is possible that the relationship between IAD scores and final grades varies by instructor, and even by class taught by the same instructor. Instructors may have different styles or abilities, and dynamics between students and instructor may also differ from class to class. The levels of the variable *instructor* (that is, the instructors themselves) are not fixed categories of individual interest in this analysis. Therefore, instead of coding the instructors using dummy variables (10 of them would be needed given, that there are 11 instructors), the effect of this relatively large number of instructors can be modeled using random coefficients in a hierarchical linear model[5].

In order to introduce the idea of a hierarchical linear model, one can consider first the not so practical alternative of fitting a regression line for each of the 32 classes in this data set. Thus, if two of these lines were to be compared, the line with the higher intercept would be indicative of a higher numerical final grade for the corresponding class. Also, if both lines had a positive slope, the line

with the larger slope would be indicative of a stronger effect of IAD scores on final grades. Now out of all of these 32 lines, lines with the largest intercepts and smallest slopes (high final grades that do not depend much on IAD scores) could be indicative of a lenient instructor, or also of a very effective instructor whose students' high grades do not depend much on IAD scores. Further interpretations could be drawn from other combinations of intercepts' and slopes' relative magnitudes.

Given the impracticality of fitting a line for each class, a hierarchical linear model could be more effective in this situation. A simple model of this kind assumes that each classroom j (level-2 unit) is a random sample from a population of classrooms whose effects U_{0j} on final grades, after adjusting for the effect of IAD scores, are random variables with mean zero and common variance τ_0^2 (population between-group variance). The model also assumes that each student i (level-1 unit) within a classroom j is a random sample from a population of students whose effects e_{ij} on final grades, after adjusting for the effect of IAD scores, are random variables with mean zero and common variance (population within-group variance) σ^2 within each classroom. A final assumption linking these variance components is that the random effects U_{0j} are independent of the random effects e_{ij}. Thus the final grades of a student i in classroom j, y_{ij}, can be represented linearly in terms of these random effects and the student's cluster-centered IAD score $(x_{ij} - \bar{x}_{.j})$ as described in the following equation:

$$y_{ij} = \gamma_{00} + \gamma_{10}(x_{ij} - \bar{x}_{.j}) + U_{0j} + e_{ij} \qquad\qquad \textbf{Equation 7.}$$

In terms of the notation used by Snijders and Bosker[5] this equation can be seen as a model that allows for classroom-specific regression models $y_{ij} = \beta_0 + \beta_1(x_{ij} - \bar{x}_{.j}) + e_{ij}$ that share the same slope, $(\beta_1 = \gamma_{10})$, but permits separate random intercepts for each classroom j $(\beta_0 = \gamma_{00} + U_{0j})$. As the reader may have surmised, in order to allow for these models to also have classroom-specific slopes, one would need to add a third set of random effects U_{1j} $(\beta_1 = \gamma_{10} + U_{1j})$ for each classroom j, which would be independent of the other two sets of random effects, and would also be assumed to be random variables with mean zero and variance τ_1^2. The resulting model would be:

$$y_{ij} = (\gamma_{00} + U_{0j}) + (\gamma_{10} + U_{1j})(x_{ij} - \bar{x}_{.j}) + e_{ij} \qquad\qquad \textbf{Equation 8.}$$

Table 8 below presents the summary of model (7) when estimating its parameters with the statistical software HLM7. A p-value of the t-test less than 0.001 for the variance component τ_0^2 of the random intercept indicates that the classrooms do differ in their average final grades. The coefficients for the fixed effects of the intercept and the slope are very close to those obtained with the simple regression model (see **Table 3**), which does not account for the clustering effect of the classrooms; however, the standard error for the intercept of the hierarchical model (0.778995) is almost twice that of the simple regression model (0.4670). This is an indication that these two estimates are rather different from a statistical point of view. In the words of Snijder and Bosker5, the simple regression model (3) "produces an over-optimistic impression of the precision of this estimate, and illustrates the lack of trustworthiness of using simple regression modeling for estimates of multilevel data." Rounding the estimates in **Table 8** to two decimals, a low average final grade classroom (bottom 2.5 percent) has a value of U_{0j} of about 2 standard deviations below the mean, so that it will have a regression line:

$$y_{ij} = 80.83 - 2(0.78) + 0.72IAD = 79.27 + 0.72IAD \qquad \textbf{Equation 9.}$$

This regression line differs slightly from that of a typical high average final grade classroom (upper 2.5 percent):

$$y_{ij} = 80.83 - 2(0.78) + 0.72IAD = 82.39 + 0.72IAD \qquad \textbf{Equation 10.}$$

Fixed Effect	Degrees of Freedom	Coefficient	Standard Error	t-value	Pr (> | t |)
Intercept γ_0	31	80.829332	0.778995	103.7611	< 0.001
Slope γ_1	490	0.719604	0.11902	6.431	< 0.001
Random Effect	**Degrees of Freedom**	**Estimate of Variance Component**	**Estimate of Standard Deviation τ_0**	**Chi Square**	**p-value**
Var of U_{0j} (τ_0^2)	31	12.18985	3.49149	86.97264	< 0.001
Var of e_{ij} (σ^2)		110.86590	10.52929		

Table 8. Summary for the hierarchical regression model in **Equation 7**.

The difference between the intercepts of lines (9) and (10), although *statistically* different, presents no *practical* difference, since the intercepts of these two lines differ (point estimates for the population average numerical final scores of classes in the 2.5th and 97.5th percentiles, respectively) by barely half a letter grade.

The hierarchical regression model (8) that allows for different slopes ($\gamma_{10} + U_{1j}$) produced no statistical difference between the slopes of the regression lines of the classrooms (p-value of t-test for the null hypothesis, var (U_{0j}) = τ_1^2 = 0, greater than 0.5) as shown in the summary of this model in **Table 9**.

Fixed Effect	Degrees of Freedom	Coefficient	Standard Error	t-value	Pr (> | t |)
Intercept γ_0	31	80.838120	0.779788	103.667	< 0.001
Slope γ_1	490	0.726215	0.114033	6.368	< 0.001
Random Effect	**Degrees of Freedom**	**Estimate of Variance Component**	**Estimate of Standard Deviation τ_0**	**Chi Square**	**p-value**
Var of U_{0j} (τ_0^2)	31	12.2468	3.49954	87.14417	< 0.001
Var of U_{1j} (τ_1^2)	31	0.01423	0.11928	29.16380	> 0.5
Var of e_{ij} (σ^2)		110.64026	10.51857		

Table 9. Summary for the hierarchical regression model in **Equation 8**.

3.2. Logistic Regression Analysis

As an alternative to converting final letter grades to approximate percentages, a logistic regression equation can be obtained to model the probability of passing the course using IAD scores as a predictor variable. This requires recoding the passing grades with a 1 and the failing grades with a 0. If the probability of passing the course given an IAD score x_i is denoted as $\pi(x_i)$, the log-odds (logarithm of the odds) or logit of these probabilities, $\text{logit}[\pi(x_i)] = log[\pi(x_i)(1-\pi(x_i))]$, can be predicted as a linear function of the IAD scores.

If passing the course is defined as obtaining a grade of at least D-, the logistic regression equation when using the entire data set is given by:

$$log\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right) = -0.37117 + 0.08958x_i \qquad \textbf{Equation 11.}$$

Whereas if passing the course is defined of obtaining a grade of at least a C-, the equation is:

$$log\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right) = -1.48029 + 0.09706x_i \qquad \textbf{Equation 12.}$$

The summaries of models (11) and (12), listed in **Tables 10** and **11**, respectively, indicate a significant relationship between the IAD scores and the probability of passing the course (p-values of z-tests for slopes less than 0.01). However, the null hypothesis that the intercept is zero cannot be rejected in both of the models. This lack of significance for the intercepts has no intrinsic value in this analysis since there are no students who enroll in the course with an IAD score of zero. The Chi-square tests with 1 degree of freedom (d.f.) at the bottom of these tables are goodness of fit tests. For both models, this test rejects the null hypothesis that the fit of the model that uses the IAD as a predictor is no better than the model with just an intercept (null model).

	Estimate	Standard Error	z–value	$Pr(>|z|)$
Intercept	-0.37117	1.05491	-0.352	0.72495
Slope	0.08958	0.03405	2.631	< 0.01
Chi-square = 7.2, d.f. = 1, p < 0.01				

Table 10. Summary for the logistic regression **Equation 11** if passing grade is at least D-.

	Estimate	Standard Error	z–value	$Pr(>|z|)$
Intercept	-1.48029	.078449	-1.887	0.059168
Slope	0.09706	0.02511	3.865	< 0.001
Chi-square = 15.91, d.f. = 1, p < 0.0001				

Table 11. Summary for the logistic regression **Equation 12** if passing grade is at least C-.

When solving for the predicted probabilities $\pi(x_i)$ in terms of the intercept β_0, and the slope β_1 in models (11) and (12), the resulting equation is:

$$\pi(x_i) = \left(\frac{\exp(\beta_0 + \beta_1 x_i)}{1 + \exp(\beta_0 + \beta_1 x_i)}\right) \qquad \textbf{Equation 13.}$$

Tables 12 and 13 list the predicted probabilities, for the given IAD scores, of passing the course with a D- and a C-, respectively. These tables also list the 95% confidence intervals of these probabilities.

IAD	Probability	CI: Lower	CI:Upper	IAD	Probability	CI: Lower	CI:Upper
18	0.7757727	0.5850923	0.8946066	32	0.923809	0.8980247	0.9434841
19	0.7909688	0.6214484	0.8971406	33	0.9298794	0.9037926	0.9492896
20	0.8053934	0.6563261	0.8996857	34	0.9354999	0.9084203	0.9549692
21	0.8190504	0.6894004	0.9022551	35	0.9406986	0.9122341	0.9603329
22	0.8319489	0.7204120	0.9048659	36	0.9455028	0.9154722	0.965269
23	0.844103	0.7491678	0.9075414	37	0.9499384	0.9182990	0.9697291
24	0.8555307	0.7755373	0.9103131	38	0.9540305	0.9208251	0.9737075
25	0.8662534	0.7994422	0.9132239	39	0.9578031	0.9231254	0.9772238
26	0.8762954	0.8208449	0.9163327	40	0.9612785	0.9252513	0.980311
27	0.8856829	0.8397333	0.9197179	41	0.9644784	0.9272387	0.9830079
28	0.8944438	0.8561118	0.9234769	42	0.9674228	0.9291132	0.9853549
29	0.9026072	0.8700039	0.9277129	43	0.9701307	0.9308935	0.9873916
30	0.9102026	0.8814821	0.9324961	44	0.9726198	0.9325935	0.9891547
31	0.9172599	0.8907203	0.9378042	45	0.9749069	0.9342239	0.9906783

Table 12. Predicted Probabilities and Confidence Intervals for passing with grade of at least D-.

The confidence interval in **Table 12** for the current minimum required IAD score of 32 indicates that one can be 95% confident that at the most 10% (1 – 0.898 = 0.102) of students with and IAD score of 32 are expected to fail the class with a grade of F. T – 0.802 = 0.198) if a C- is required to pass the course (see **Table 13**).

IAD	Probability	CI: Lower	CI:Upper	IAD	Probability	CI: Lower	CI:Upper
18	0.5663090	0.3997442	0.7191292	32	0.8355744	0.8016251	0.8646944
19	0.5899766	0.4345689	0.7292797	33	0.8484791	0.8141652	0.8774111
20	0.6132346	0.4699295	0.7392909	34	0.8605401	0.8249142	0.8898846
21	0.6359874	0.5054408	0.749175	35	0.8717861	0.8343183	0.9017783
22	0.6581479	0.5407022	0.7589502	36	0.8822493	0.8427208	0.9128704
23	0.6796388	0.5753120	0.7686434	37	0.8919644	0.8503639	0.9230462
24	0.7003937	0.6088788	0.7782927	38	0.9009680	0.8574136	0.9322693
25	0.7203572	0.6410306	0.7879527	39	0.9092975	0.8639835	0.9405539
26	0.7394853	0.6714189	0.7977000	40	0.9169910	0.8701523	0.9479448
27	0.7577450	0.6997189	0.8076407	41	0.9240864	0.8759761	0.9545035
28	0.7751141	0.7256294	0.8179135	42	0.9306212	0.8814959	0.960299
29	0.7915804	0.7488827	0.828679	43	0.936632	0.8867425	0.9654028
30	0.807141	0.7692857	0.8400763	44	0.9421544	0.8917399	0.9698848
31	0.8218014	0.7868013	0.8521355	45	0.9472227	0.8965071	0.9738117

Table 13. Predicted Probabilities and Confidence Intervals for passing with grade of at least C-.

4. CONCLUSIONS

All models in this analysis show that there is a significant positive relationship between IAD scores and final grades for students registering in the introductory statistics course Stat 1 at CSUS. The regression models indicate that on average an increase of one point in the IAD score increases the numerical final grade by 0.74 of a percentage point. The hierarchical linear model showed that this relationship of IAD scores and final grades is independent of the instructors.

While the linear regressions models produced confidence and prediction intervals that were too wide to be helpful, the logistic regression model did produce more narrow and informative confidence intervals. The confidence intervals indicate that for students with IAD scores of at least 32, the maximum failing rate with a grade of F, is on average 10%. This percentage increases to 20% for students who need to pass the class with a grade of at least C-. The increasing bounds of the confidence intervals show that these failing rates increase as the IAD scores decrease. This model can then be used by the department to adjust the minimum IAD score based on its goals in terms of failing rates for Stat 1.

Therefore, the results of this study supports the use of the IAD test as a tool to screen students registering in Stat 1. The IAD test is not necessarily testing students in statistical concepts, but it may indeed be measuring a level of abstract thinking required for success in Stat 1. IAD scores are not predictive of particular final grades in Stat 1, but they are useful in predicting the failing, and passing, rates in this course. Current efforts at CSUS and other universities to expedite students' time-to-degree may find this study useful in helping them decide the risks of enrolling underprepared students in Stat 1.

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PRESS SUMMARY

The use of the intermediate algebra diagnostic (IAD) test as a screening tool for a statistics course (Stat 1) in the Department of Mathematics and Statistics at California State University Sacramento has been questioned by some faculty members and students at this university. The statistical analysis in this study shows that higher IAD scores are indeed related to higher final grades in Stat 1, even after adjusting for different instructors when using a sample of over 550 students who have taken this course. Inferences were also made in this study to predict passing grades and passing rates in Stat 1 when using IAD scores as predictors.