Model 2

This Model uses boundary & initial conditions from excel starting at **t=0.3** for trial one and **t=0.8** for trial two.
Additionally the initial conditions are **linear** fit, and **thermistors are shifted** such that $R_6 = 7$ cm.

```
Clear["Global`*"]
```

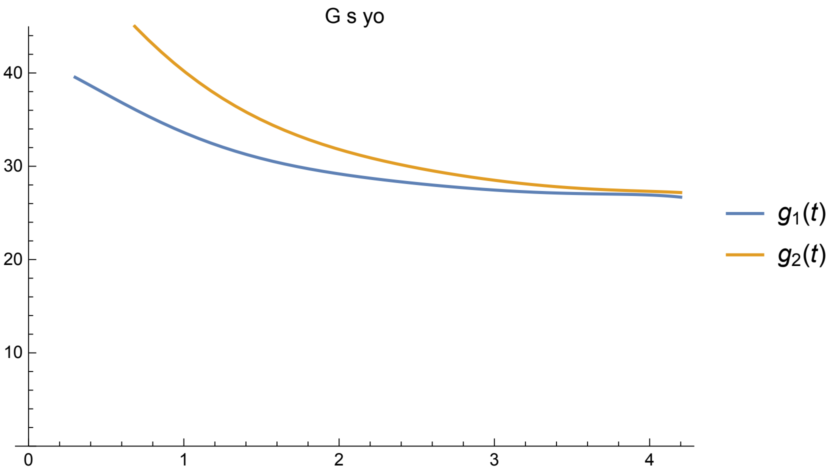
Set up the input variables and functions

```
In[246]:= (* Graph Bounds : NOT PART OF THE PROBLEM *) d1[r_] = (r - R1) × 10^10;
d2[r_] = (r - R6) × 10^10;
κ = (3.412 × 10^-5) 60;        (* Thermal Diffusiivity : m^2/s *)

R1 = 0.010;             (* Inner Radius *)
R2 = 0.020;
R3 = 0.040;
R4 = 0.050;
R5 = 0.060;
R6 = 0.070;             (* Outer Radius *)




(* Set up boundary conditions for each trial *)
g1[t_] = 41.889 - 6.1995 t - 7.6973 t^2 + 8.4423 t^3 - 3.4099 t^4 + 0.6391 t^5 - 0.0461 t^6;
(* Boundary Condition at Ri  || R^2= *)
(*g2[t_]=
  -2.3375+144.35 t-179.13 t^2+103.76t^3-31.565 t^4+4.8792 t^5-0.3021 t^6;    *)
(* Boundary Condition at Ri  || R^2= *)
g2[t_] = 59.484 - 24.435 t + 2.8109 t^2 + 4.1342 t^3 - 2.1891 t^4 + 0.4346 t^5 - 0.0314 t^6;



h1[t_] = 32.671 - 55.093 t + 69.986 t^2 - 45.935 t^3 + 15.945 t^4 - 2.7714 t^5 + 0.1897 t^6;
(* Boundary Condition at Ro  || R^2= *)
(*h2[t_]=
  259.76-632.44 t+658.78 t^2-353.08 t^3+102.55 t^4-15.332 t^5+0.9247 t^6;     *)
(* Boundary Condition at Ro  || R^2= *)
h2[t_] = 165.64 - 375.47 t + 381.78 t^2 - 201.41 t^3 + 57.822 t^4 - 8.5651 t^5 + 0.5125 t^6;
```

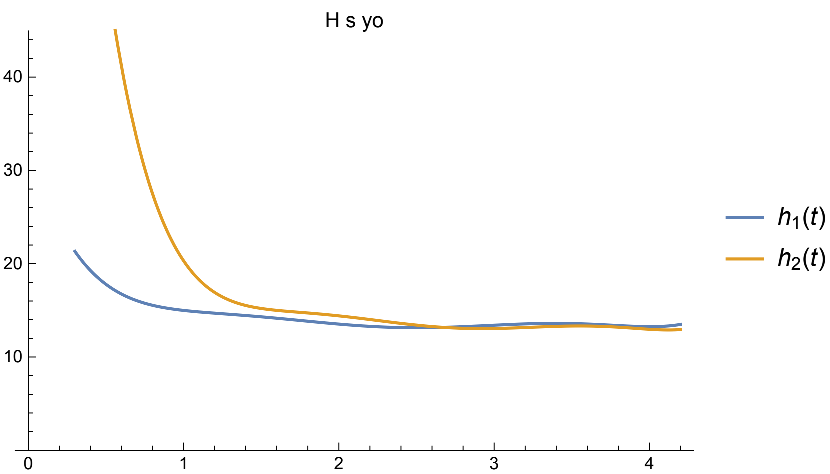
```
(* Set up the initial conditions functions and build their transforms *)
V1[r_] = 43.77 - 0.2709 (1000 r);  (* Original IC || R^2= *)
(*V2[r_]=44.838-0.3085(1000r);*)
(*V2[r_]=41.485-0.1312(1000r); *) (* Original IC || R^2= *)

V2[r_] = -0.0046 (1000 r)^2 + 0.0551 (1000 r) + 39.776;

V̂1[r_] = V1[r] - ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) × g1[0.3] - ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) × h1[0.3];
(* Transformed IC *)
V̂2[r_] = V2[r] - ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) × g2[0.8] - ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) × h2[0.8];
(* Transformed IC *)
```

```
(* Build the Forcing Functions and their transforms *)
F1[r_, t_] = 0;          (* Original Forcing *)
F2[r_, t_] = 0;          (* Original Forcing *)
F̂1[r_, t_] = F1[r, t] - ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) D[g1[t], t] -
    ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) D[h1[t], t]; (* Transformed Forcing *)
F̂2[r_, t_] = F2[r, t] - ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) D[g2[t], t] - ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) D[h2[t], t];
(* Transformed Forcing *)


Plot[{g1[t], g2[t]}, {t, 0.3, 4.2}, PlotRange → {0, 45},
 PlotLabel → "G s yo", PlotLegends → "Expressions"]

Plot[{h1[t], h2[t]}, {t, 0.3, 4.2}, PlotRange → {0, 45},
 PlotLabel → "H s yo", PlotLegends → "Expressions"]
```

Out[266]=



Out[267]=

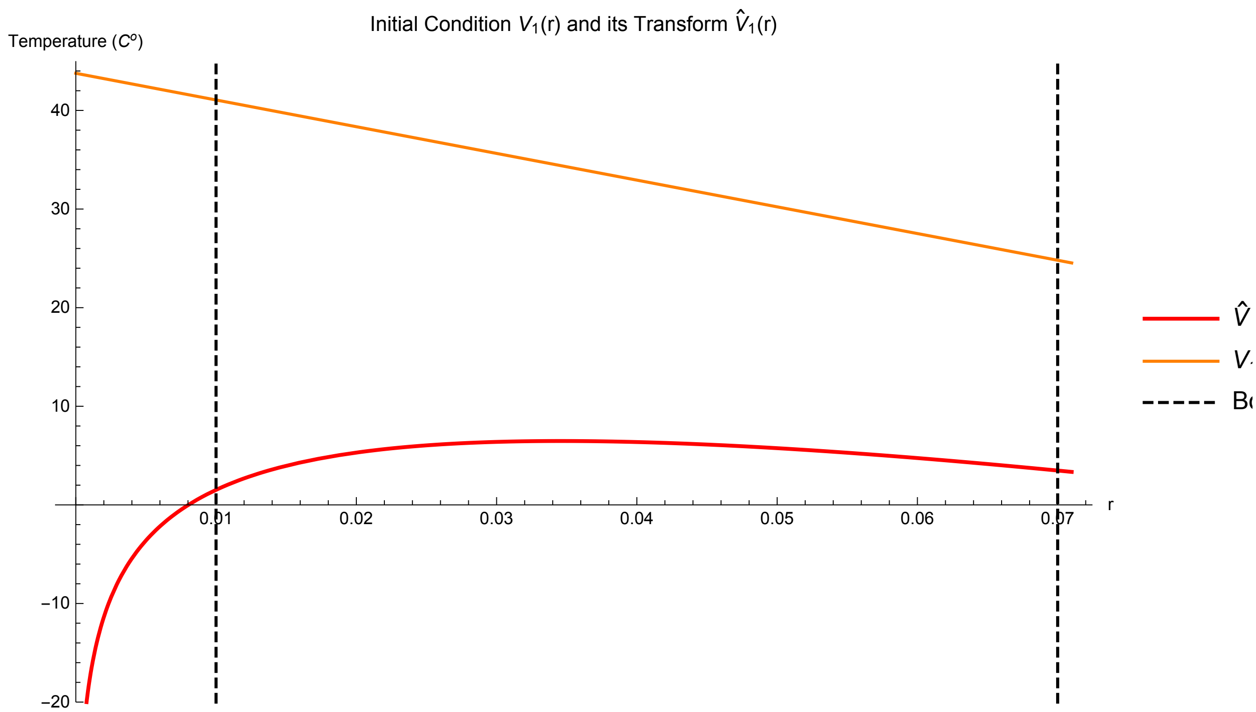


Plot Initial Condition Info

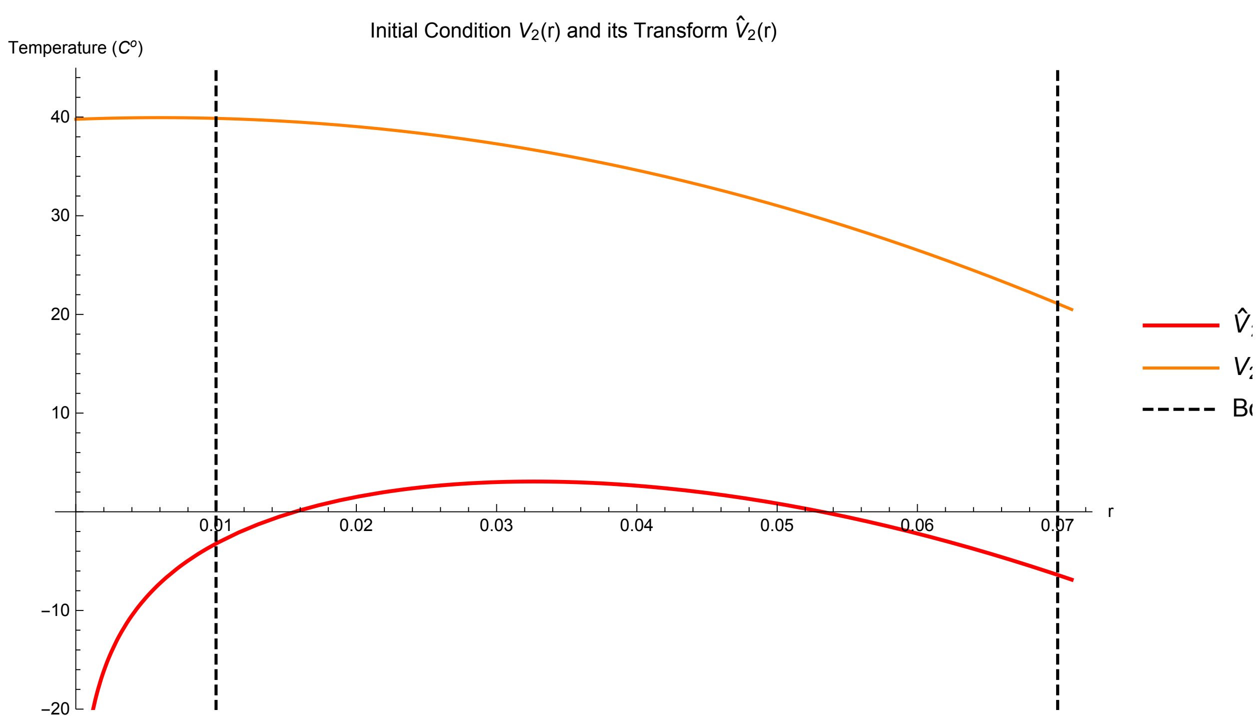
In[268]:=

```
(* Plot the initial condition information  *)
Plot[{V̂₁[r], V₁[r], d1[r], d2[r]}, {r, R₁ - 10^-2, R₆ + 10^-3}, PlotRange → {-20, 45},
 PlotStyle → {{Red, Thick}, {Orange}, {Black, Dashed}, {Black, Dashed}},
 ImageSize → Large, PlotLabel → "Initial Condition V₁(r) and its Transform V̂₁(r)",
 AxesLabel → {"r", "Temperature (C°)"},
 PlotLegends → {"V̂₁(r)", "V₁(r)", "Boundaries"}]

Plot[{V̂₂[r], V₂[r], d1[r], d2[r]}, {r, R₁ - 10^-2, R₆ + 10^-3}, PlotRange → {-20, 45},
 PlotStyle → {{Red, Thick}, {Orange}, {Black, Dashed}, {Black, Dashed}},
 ImageSize → Large, PlotLabel → "Initial Condition V₂(r) and its Transform V̂₂(r)",
 AxesLabel → {"r", "Temperature (C°)"},
 PlotLegends → {"V̂₂(r)", "V₂(r)", "Boundaries"}]
```

Out[268]=



Out[269]=



Find m_n Values

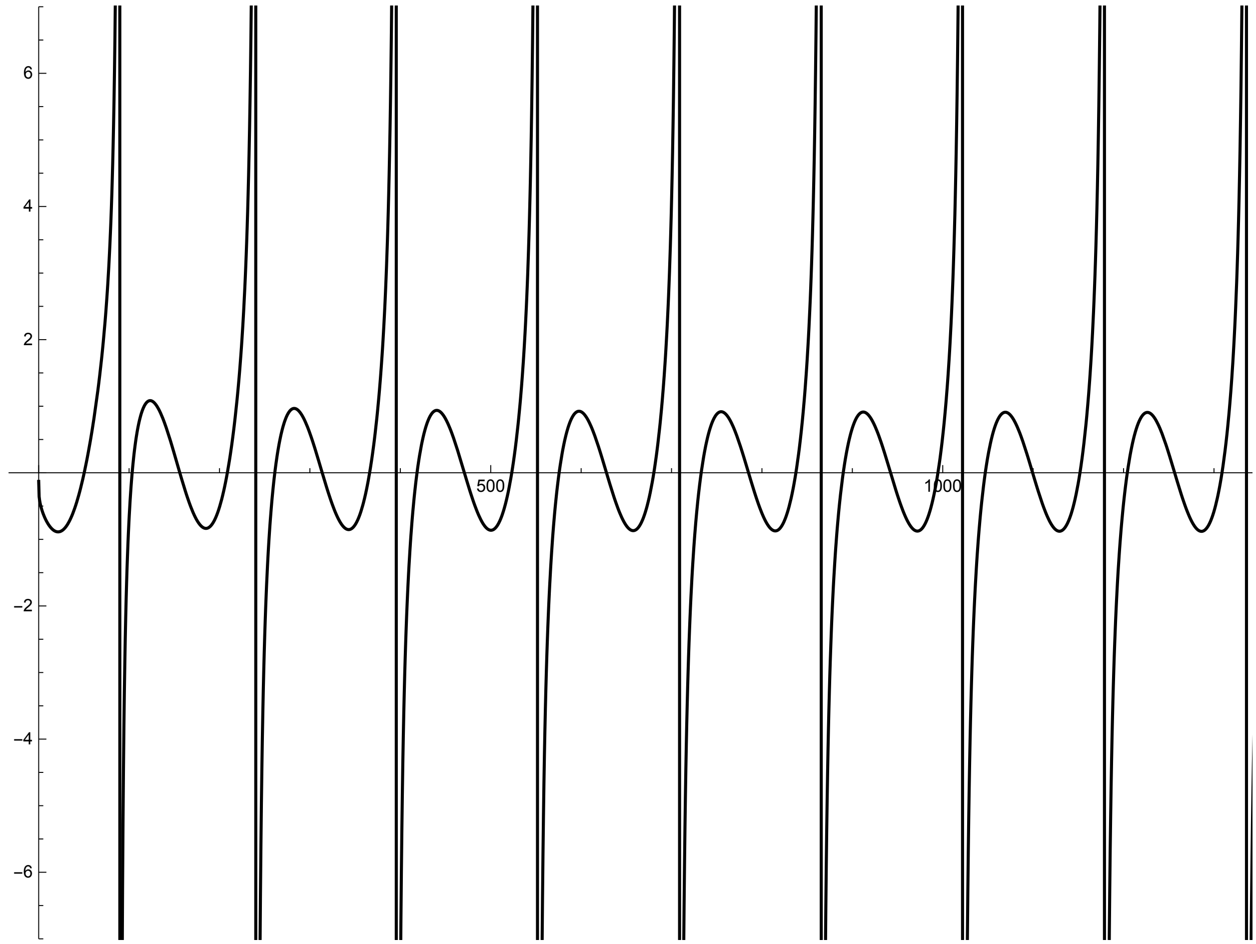
In[270]:=

```
(* Create function which has roots
  that are the roots of the Bessel eigen-functions *)
```

$$M[m\_] = \frac{\text{BesselY}[0, (m R_6)]}{\text{BesselY}[0, (m R_1)]} - \frac{\text{BesselJ}[0, (m R_6)]}{\text{BesselJ}[0, (m R_1)]};$$

```
(* The roots of this EQn satisfy the Boundary Conditions *)



(* Plot Function to identify roots *)
Plot[M[m], {m, 0, 1600}, PlotStyle → Black, PlotRange → {-7, 7}]


(* Determine numerical Eigen Values *)
FindRoot[M[m], {m, 0050}] FindRoot[M[m], {m, 0105}]
 FindRoot[M[m], {m, 0155}] FindRoot[M[m], {m, 0210}] FindRoot[M[m], {m, 260}]
FindRoot[M[m], {m, 0310}] FindRoot[M[m], {m, 0370}] FindRoot[M[m], {m, 0420}]
 FindRoot[M[m], {m, 0480}] FindRoot[M[m], {m, 0520}]

FindRoot[M[m], {m, 0570}] FindRoot[M[m], {m, 0620}]
 FindRoot[M[m], {m, 0690}] FindRoot[M[m], {m, 0730}] FindRoot[M[m], {m, 0790}]
FindRoot[M[m], {m, 840}] FindRoot[M[m], {m, 900}] FindRoot[M[m], {m, 950}]
 FindRoot[M[m], {m, 1000}] FindRoot[M[m], {m, 1050}]

FindRoot[M[m], {m, 1100}] FindRoot[M[m], {m, 1150}]
 FindRoot[M[m], {m, 1205}] FindRoot[M[m], {m, 1260}] FindRoot[M[m], {m, 1310}]
FindRoot[M[m], {m, 1360}] FindRoot[M[m], {m, 1410}] FindRoot[M[m], {m, 1460}]
 FindRoot[M[m], {m, 1520}] FindRoot[M[m], {m, 1570}]


mn = {50.32451610924386, 103.3822580997227, 156.09380328929868, 208.66292108233847,
   261.16064951493075, 313.6177799968187, 366.04973666487865, 418.46506383053395,
   470.8688601991104, 523.2643483625127, 575.6536607485555, 628.0382623703335,
   680.4191917158894, 732.7972047071326, 785.1728642493102, 837.5465978748034,
   889.9187359636916, 942.2895377498468, 994.6592094241732, 1047.027916994561,
   1099.3957955882045, 1151.7629562911868, 1204.1294912520495,
   1256.4954775414446, 1308.860980106999, 1361.2260540610496, 1413.5907464702525,
   1465.9550977689396, 1518.3191428852197, 1570.6829121455937};
```

Out[271]=



Out[272]= { (m → 50.3245) (m → 103.382) (m → 156.094) (m → 208.663) (m → 261.161) }

Out[273]= { (m → 313.618) (m → 366.05) (m → 418.465) (m → 470.869) (m → 523.264) }

Out[274]= { (m → 575.654) (m → 628.038) (m → 680.419) (m → 732.797) (m → 785.173) }

Out[275]= { (m → 837.547) (m → 889.919) (m → 942.29) (m → 994.659) (m → 1047.03) }

Out[276]= { (m → 1099.4) (m → 1151.76) (m → 1204.13) (m → 1256.5) (m → 1308.86) }

Out[277]= { (m → 1361.23) (m → 1413.59) (m → 1465.96) (m → 1518.32) (m → 1570.68) }

Find B_n Values

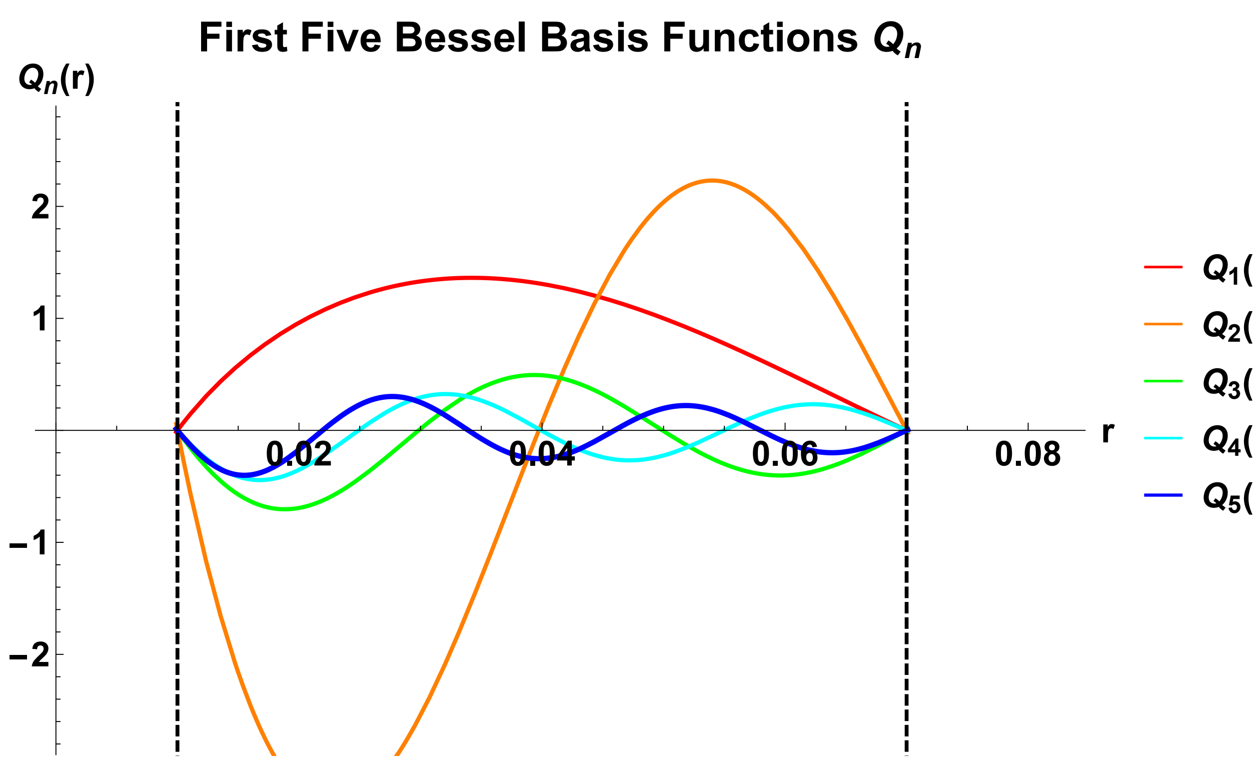
In[279]:= $B_n = -\frac{\text{BesselJ}[0, R_1\, m_n]}{\text{BesselY}[0, R_1\, m_n]};$

Define Basis Functions and Display first five Functions

In[280]:=

```
(* Basis Functions (indexed by 'n') *)
Qn[r_] = BesselJ[0, mn r] + Bn BesselY[0, mn r];


(* Create Plot of first five basis functions *)
WOW = Plot[{Qn[r][[1]], Qn[r][[2]], Qn[r][[3]], Qn[r][[4]], Qn[r][[5]], d1[r], d2[r]},
   {r, R1 - 10^-4, R6 + 10^-4}, PlotRange → {-9/2, 9/2},
   PlotStyle → {{Thickness[0.004], Red}, {Thickness[0.004], Orange},
     {Thickness[0.004], Green}, {Thickness[0.004], Cyan},
     {Thickness[0.005], Blue}, {Black, Dashed, Thick}, {Black, Dashed, Thick}},
   PlotLabel → "First Five Bessel Basis Qn", AxesLabel → {"r", "Qn(r)"},
   ImageSize → Large, LabelStyle → {Directive[Bold, Black], 18},
   PlotLegends → {"Q1(r)", "Q2(r)", "Q3(r)", "Q4(r)", "Q5(r)"}];

(* Adjust bases plot *)
AH = Plot[r, {r, 0, R6 + 0.013}, PlotStyle → {White}, PlotRange → {-2.9, 2.9},
   PlotLabel → "First Five Bessel Basis Functions Qn", LabelStyle →
    {Directive[Bold, Black], 18}, AxesLabel → {"r", "Qn(r)"}, ImageSize → Large];

(* Display Functions *)
Show[AH, WOW]
```

Out[283]=



Find L_n Values

In[284]:=

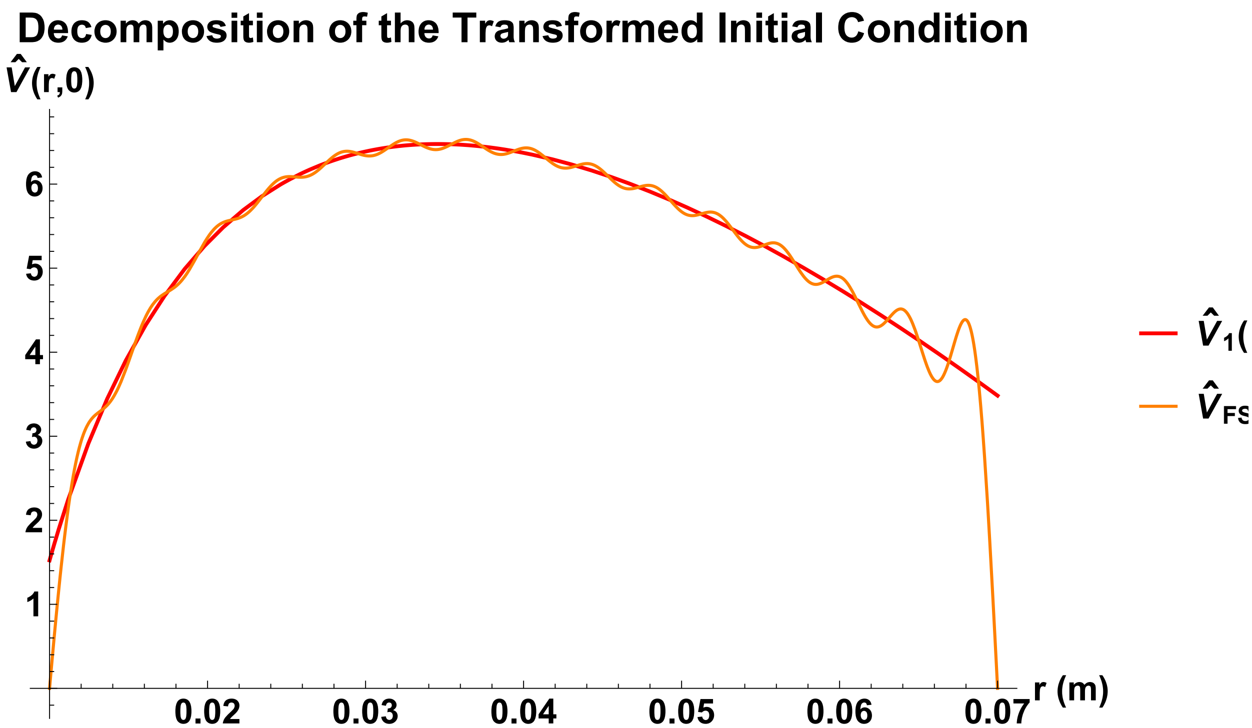
```
Lₙ = NIntegrate[r × Qₙ[r]^2, {r, R₁, R₆}];
```

Decompose Initial Conditions (Find A_ns)

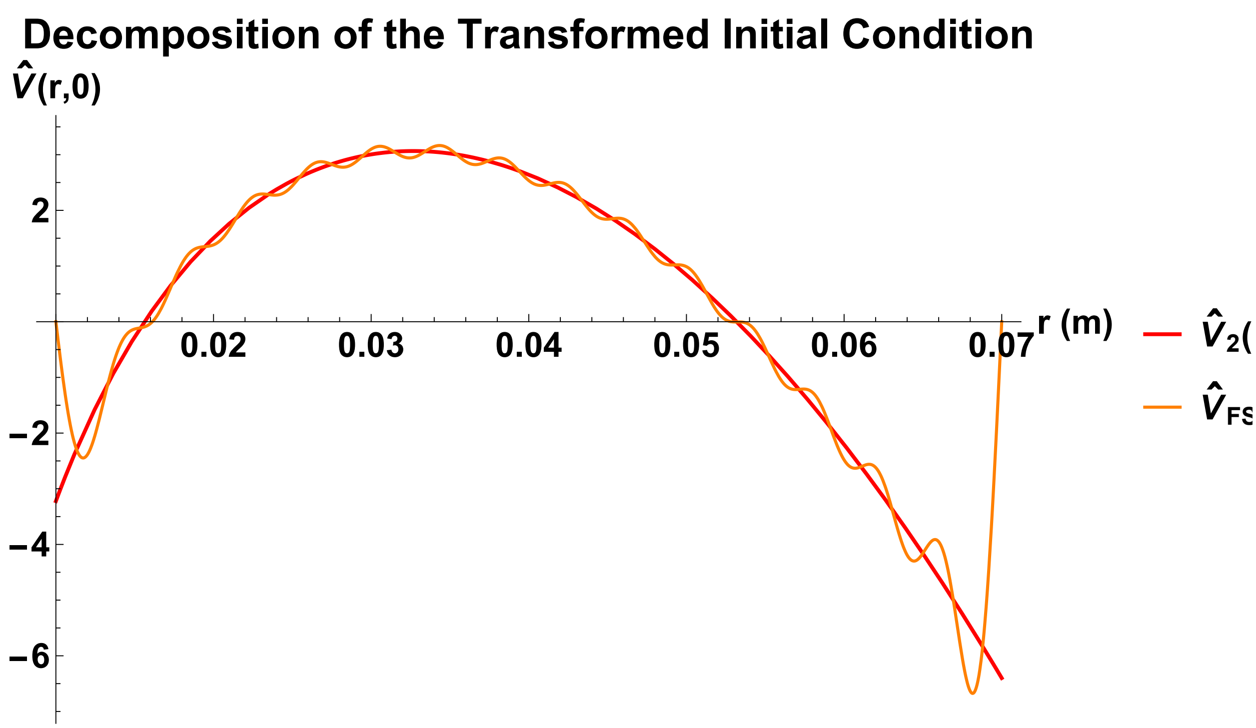
In[285]:=

```
(* Decompose and Plot first trial *)
A1n = (1/Ln) NIntegrate[r × V̂1[r] × Qn[r] , {r, R1, R6}];
V̂FS1[r_] = Sum[A1n[[j]] Qn[r][[j]], {j, 1, 30}];
Plot[{V̂1[r], V̂FS1[r]}, {r, R1, R6}, PlotStyle → {{Red, Thick}, Orange},
 PlotLabel → "Decomposition of the Transformed Initial Condition",
 AxesLabel → {"r  (m)", "V̂(r,0)"}, PlotLegends → {"V̂1(r)", "V̂FS1(r)"},
 ImageSize → Large, LabelStyle → {Directive[Bold, Black], 18}]


(* Decompose and Plot seceond trial *)
A2n = (1/Ln) NIntegrate[r × V̂2[r] × Qn[r] , {r, R1, R6}];
V̂FS2[r_] = Sum[A2n[[j]] Qn[r][[j]], {j, 1, 30}];
Plot[{V̂2[r], V̂FS2[r]}, {r, R1, R6}, PlotStyle → {{Red, Thick}, Orange},
 PlotLabel → "Decomposition of the Transformed Initial Condition",
 AxesLabel → {"r  (m)", "V̂(r,0)"}, PlotLegends → {"V̂2(r)", "V̂FS2(r)"},
 ImageSize → Large, LabelStyle → {Directive[Bold, Black], 18}]
```

Out[287]=



Out[290]=



Decompose Forcing Functions (Find $A'_n(t)$s)

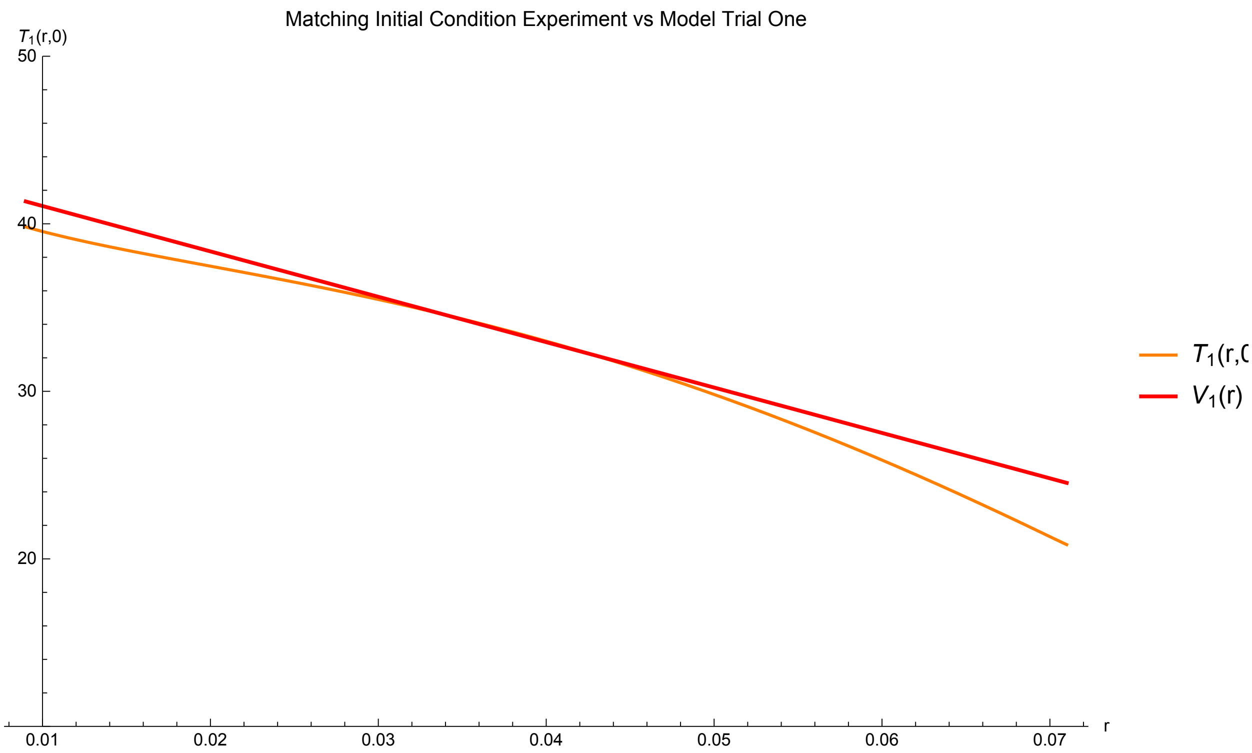
In[291]:=
```
(* Decompose Forcing Function and derive final solution for trial one *)
a1n[t_] = -1/Ln (NIntegrate[r × ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) × Qn[r], {r, R1, R6}] D[g1[t], t] +
      NIntegrate[r × ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) × Qn[r], {r, R1, R6}] D[h1[t], t]) e^(mn^2 κ t);
A1n  = 1/Ln  NIntegrate[r × V̂1[r] × Qn[r]  , {r, R1, R6}];
x1n[t_] = Integrate[a1n[t], t];
y1n[t_] = x1n[t] - x1n[0];
z1[r_, t_] = (Sum[((y1n[t][[j]] + A1n[[j]]) Qn[r][[j]] × e^(-(mn[[j]])^2 κ t)), {j, 1, 30}]);
T1[r_, t_] = z1[r, t] + ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) g1[t] + ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) h1[t];


(* Decompose Forcing Function and derive final solution for trial two *)
a2n[t_] = -1/Ln (NIntegrate[r × ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) × Qn[r], {r, R1, R6}] D[g2[t], t] +
      NIntegrate[r × ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) × Qn[r], {r, R1, R6}] D[h2[t], t]) e^(mn^2 κ t);
A2n  = 1/Ln  NIntegrate[r × V̂2[r] × Qn[r]  , {r, R1, R6}];
x2n[t_] = Integrate[a2n[t], t];
y2n[t_] = x2n[t] - x2n[0];
z2[r_, t_] = (Sum[((y2n[t][[j]] + A2n[[j]]) Qn[r][[j]] × e^(-(mn[[j]])^2 κ t)), {j, 1, 30}]);
T2[r_, t_] = z2[r, t] + ((Log[R6] - Log[r])/(Log[R6] - Log[R1])) g2[t] + ((Log[r] - Log[R1])/(Log[R6] - Log[R1])) h2[t];
```

Verify completed Models Satisfy Initial Conditions

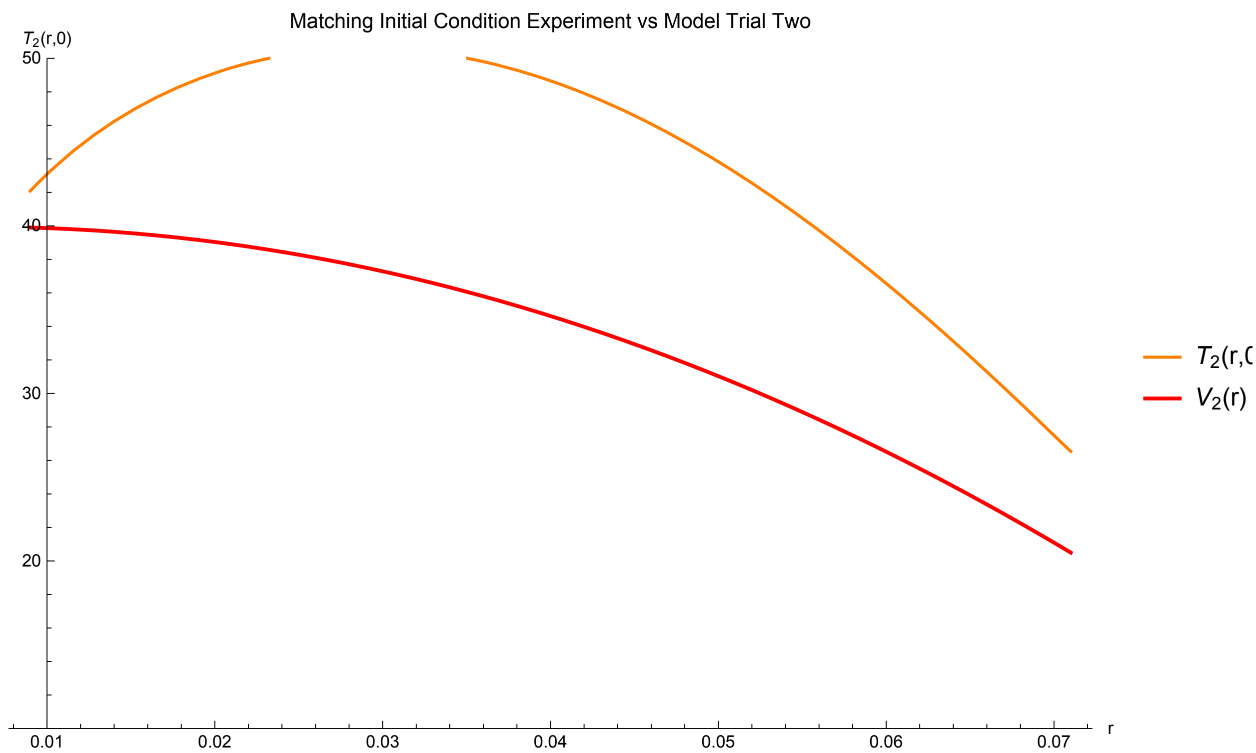
In[303]:=

```
(* Verify For Trial One *)
Plot[{T1[r, 0.3], V1[r]}, {r, R1 - 10^-3, R6 + 10^-3},
 PlotRange → {10, 50}, PlotStyle → {Orange, {Red, Thick}},
 PlotLabel → "Matching Initial Condition Experiment vs Model Trial One",
 AxesLabel → {"r", "T1(r,0)"},
 PlotLegends → {"T1(r,0)      Model", "V1(r)         Experiment"}, ImageSize → Large]

(* Verify For Trial Two *)
Plot[{T2[r, 0.8], V2[r]}, {r, R1 - 10^-3, R6 + 10^-3},
 PlotRange → {10, 50}, PlotStyle → {Orange, {Red, Thick}},
 PlotLabel → "Matching Initial Condition Experiment vs Model Trial Two",
 AxesLabel → {"r", "T2(r,0)"},
 PlotLegends → {"T2(r,0)      Model", "V2(r)         Experiment"}, ImageSize → Large]
```

Out[303]=



Out[304]=



■ DATA

In[305]:=

```
Time = {4.2`, 4.`, 3.9`, 3.8`, 3.6`, 3.5`, 3.4`, 3.2`, 3.1`,
   3.`, 2.8`, 2.7`, 2.6`, 2.4`, 2.3`, 2.2`, 2.`, 1.9`, 1.8`, 1.6`, 1.5`,
   1.4`, 1.2`, 1.1`, 1.`, 0.8`, 0.7`, 0.6`, 0.4`, 0.3`, 0.2`, 0.`};
Time2 = {4.2`, 4.`, 3.9`, 3.8`, 3.6`, 3.5`, 3.4`, 3.2`, 3.1`, 3.`,
   2.8`, 2.7`, 2.6`, 2.4`, 2.3`, 2.2`, 2.`, 1.9`, 1.8`, 1.6`, 1.5`,
   1.4`, 1.2`, 1.1`, 1.`, 0.8`, 0.7`, 0.6`, 0.4`, 0.3`, 0.2`, 0.`};

Th1aMAT = {{4.2`, 26.9`}, {4.`, 27.`}, {3.9`, 27.`}, {3.8`, 27.1`},
   {3.6`, 27.1`}, {3.5`, 27.2`}, {3.4`, 27.2`}, {3.2`, 27.3`},
   {3.1`, 27.4`}, {3.`, 27.5`}, {2.8`, 27.7`}, {2.7`, 27.8`}, {2.6`, 28.`},
   {2.4`, 28.2`}, {2.3`, 28.5`}, {2.2`, 28.7`}, {2.`, 29.1`}, {1.9`, 29.5`},
   {1.8`, 29.9`}, {1.6`, 30.4`}, {1.5`, 30.9`}, {1.4`, 31.5`},
```

```
    {1.2`, 32.1`}, {1.1`, 32.9`}, {1.`, 33.7`}, {0.8`, 34.7`}, {0.7`, 35.9`},
    {0.6`, 37.3`}, {0.4`, 38.7`}, {0.3`, 39.4`}, {0.2`, 39.2`}, {0.`, 39.`}};
Th2aMAT = {{4.2`, 21.5`}, {4.`, 21.6`}, {3.9`, 21.6`}, {3.8`, 21.6`},
    {3.6`, 21.6`}, {3.5`, 21.7`}, {3.4`, 21.8`}, {3.2`, 21.8`},
    {3.1`, 21.8`}, {3.`, 21.9`}, {2.8`, 22.`}, {2.7`, 22.1`}, {2.6`, 22.2`},
    {2.4`, 22.5`}, {2.3`, 22.7`}, {2.2`, 22.9`}, {2.`, 23.3`}, {1.9`, 23.6`},
    {1.8`, 24.`}, {1.6`, 24.5`}, {1.5`, 25.`}, {1.4`, 25.6`}, {1.2`, 26.3`},
    {1.1`, 27.`}, {1.`, 28.`}, {0.8`, 29.1`}, {0.7`, 30.5`}, {0.6`, 32.5`},
    {0.4`, 35.5`}, {0.3`, 38.4`}, {0.2`, 38.3`}, {0.`, 38.`}};
Th3aMAT = {{4.2`, 17.5`}, {4.`, 17.6`}, {3.9`, 17.6`}, {3.8`, 17.6`},
    {3.6`, 17.7`}, {3.5`, 17.7`}, {3.4`, 17.7`}, {3.2`, 17.8`},
    {3.1`, 17.7`}, {3.`, 17.8`}, {2.8`, 17.8`}, {2.7`, 17.9`}, {2.6`, 18.`},
    {2.4`, 18.1`}, {2.3`, 18.2`}, {2.2`, 18.5`}, {2.`, 18.8`}, {1.9`, 19.`},
    {1.8`, 19.4`}, {1.6`, 19.7`}, {1.5`, 20.2`}, {1.4`, 20.6`}, {1.2`, 21.2`},
    {1.1`, 21.8`}, {1.`, 22.6`}, {0.8`, 23.5`}, {0.7`, 24.9`}, {0.6`, 26.9`},
    {0.4`, 30.2`}, {0.3`, 34.6`}, {0.2`, 35.1`}, {0.`, 34.7`}};
Th4aMAT = {{4.2`, 15.5`}, {4.`, 15.5`}, {3.9`, 15.6`}, {3.8`, 15.7`},
    {3.6`, 15.7`}, {3.5`, 15.7`}, {3.4`, 15.7`}, {3.2`, 15.7`},
    {3.1`, 15.8`}, {3.`, 15.7`}, {2.8`, 15.7`}, {2.7`, 15.7`}, {2.6`, 15.8`},
    {2.4`, 15.8`}, {2.3`, 16.`}, {2.2`, 16.1`}, {2.`, 16.4`}, {1.9`, 16.6`},
    {1.8`, 16.9`}, {1.6`, 17.2`}, {1.5`, 17.5`}, {1.4`, 18.`}, {1.2`, 18.5`},
    {1.1`, 19.`}, {1.`, 19.7`}, {0.8`, 20.4`}, {0.7`, 21.5`}, {0.6`, 23.3`},
    {0.4`, 26.4`}, {0.3`, 32.3`}, {0.2`, 34.2`}, {0.`, 33.9`}};
Th5aMAT = {{4.2`, 13.9`}, {4.`, 13.9`}, {3.9`, 14.`}, {3.8`, 14.`},
    {3.6`, 14.1`}, {3.5`, 14.`}, {3.4`, 14.1`}, {3.2`, 14.`}, {3.1`, 14.`},
    {3.`, 14.`}, {2.8`, 14.`}, {2.7`, 13.9`}, {2.6`, 14.`}, {2.4`, 14.`},
    {2.3`, 14.1`}, {2.2`, 14.2`}, {2.`, 14.4`}, {1.9`, 14.6`}, {1.8`, 14.9`},
    {1.6`, 15.1`}, {1.5`, 15.4`}, {1.4`, 15.7`}, {1.2`, 16.1`},
    {1.1`, 16.5`}, {1.`, 17.`}, {0.8`, 17.6`}, {0.7`, 18.5`}, {0.6`, 19.7`},
    {0.4`, 22.2`}, {0.3`, 28.1`}, {0.2`, 33.`}, {0.`, 32.8`}};
Th6aMAT = {{4.2`, 13.2`}, {4.`, 13.3`}, {3.9`, 13.3`}, {3.8`, 13.4`},
    {3.6`, 13.4`}, {3.5`, 13.4`}, {3.4`, 13.4`}, {3.2`, 13.4`},
    {3.1`, 13.4`}, {3.`, 13.3`}, {2.8`, 13.3`}, {2.7`, 13.3`}, {2.6`, 13.3`},
    {2.4`, 13.3`}, {2.3`, 13.3`}, {2.2`, 13.3`}, {2.`, 13.4`}, {1.9`, 13.6`},
    {1.8`, 13.7`}, {1.6`, 13.9`}, {1.5`, 14.2`}, {1.4`, 14.4`}, {1.2`, 14.7`},
    {1.1`, 15.`}, {1.`, 15.3`}, {0.8`, 15.7`}, {0.7`, 16.1`}, {0.6`, 16.8`},
    {0.4`, 18.1`}, {0.3`, 22.1`}, {0.2`, 31.6`}, {0.`, 31.3`}};

Th1bMAT = {{4.2`, 27.2`}, {4.`, 27.3`}, {3.9`, 27.4`},
    {3.8`, 27.5`}, {3.6`, 27.6`}, {3.5`, 27.8`}, {3.4`, 27.9`},
    {3.2`, 28.`}, {3.1`, 28.3`}, {3.`, 28.5`}, {2.8`, 28.9`}, {2.7`, 29.2`},
    {2.6`, 29.6`}, {2.4`, 30.1`}, {2.3`, 30.6`}, {2.2`, 31.1`}, {2.`, 31.6`},
    {1.9`, 32.3`}, {1.8`, 33.1`}, {1.6`, 33.9`}, {1.5`, 34.9`}, {1.4`, 36.1`},
    {1.2`, 37.6`}, {1.1`, 39.2`}, {1.`, 40.1`}, {0.8`, 40.`}, {0.7`, 39.8`},
    {0.6`, 39.5`}, {0.4`, 39.3`}, {0.3`, 39.2`}, {0.2`, 39.`}, {0.`, 38.7`}};
Th2bMAT = {{4.2`, 21.8`}, {4.`, 21.8`}, {3.9`, 21.9`}, {3.8`, 21.9`},
    {3.6`, 22.`}, {3.5`, 22.1`}, {3.4`, 22.2`}, {3.2`, 22.3`},
    {3.1`, 22.5`}, {3.`, 22.6`}, {2.8`, 22.9`}, {2.7`, 23.3`}, {2.6`, 23.7`},
    {2.4`, 24.2`}, {2.3`, 24.6`}, {2.2`, 25.1`}, {2.`, 25.7`}, {1.9`, 26.3`},
    {1.8`, 27.1`}, {1.6`, 28.`}, {1.5`, 29.1`}, {1.4`, 30.6`}, {1.2`, 32.6`},
    {1.1`, 35.4`}, {1.`, 38.7`}, {0.8`, 39.2`}, {0.7`, 38.9`}, {0.6`, 38.7`},
    {0.4`, 38.4`}, {0.3`, 38.1`}, {0.2`, 37.8`}, {0.`, 37.6`}};
Th3bMAT = {{4.2`, 17.7`}, {4.`, 17.7`}, {3.9`, 17.7`}, {3.8`, 17.7`},
    {3.6`, 17.8`}, {3.5`, 17.8`}, {3.4`, 17.9`}, {3.2`, 18.`}, {3.1`, 18.1`},
    {3.`, 18.2`}, {2.8`, 18.4`}, {2.7`, 18.7`}, {2.6`, 19.`}, {2.4`, 19.4`},
    {2.3`, 19.9`}, {2.2`, 20.2`}, {2.`, 20.7`}, {1.9`, 21.2`}, {1.8`, 21.8`},
    {1.6`, 22.6`}, {1.5`, 23.5`}, {1.4`, 24.7`}, {1.2`, 26.7`},
```

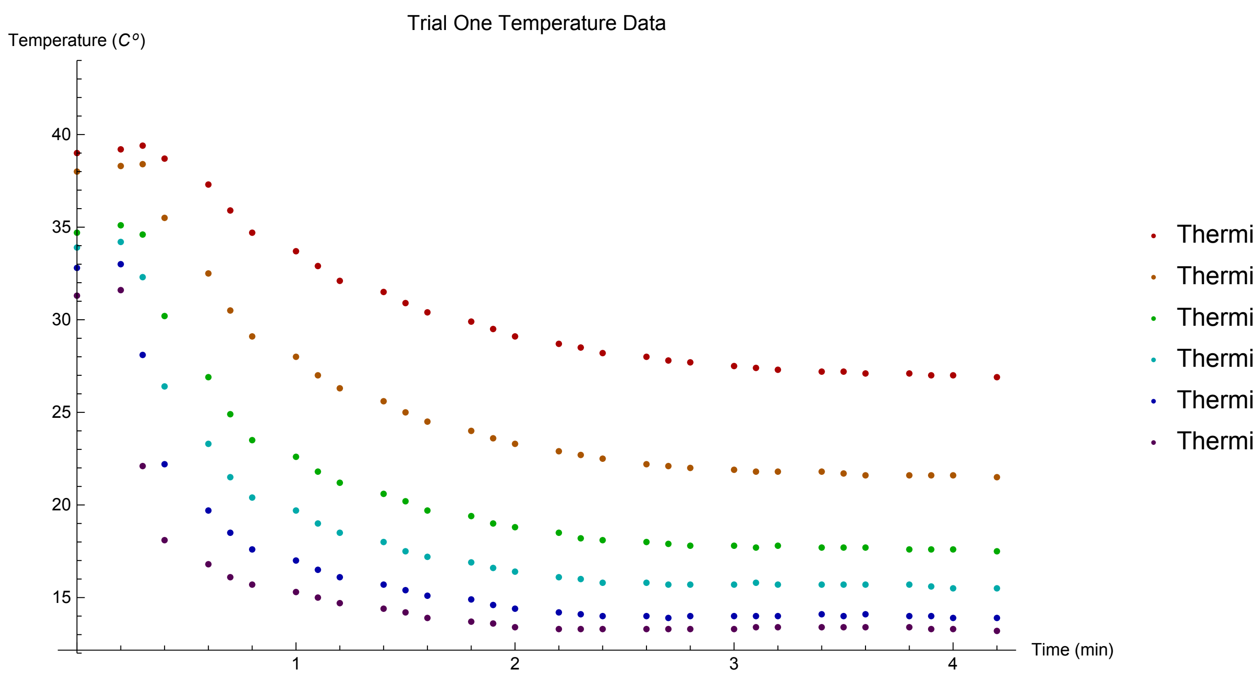
```
    {1.1`, 29.8`}, {1.`, 34.4`}, {0.8`, 35.9`}, {0.7`, 35.7`}, {0.6`, 35.4`},
    {0.4`, 35.1`}, {0.3`, 34.8`}, {0.2`, 34.6`}, {0.`, 34.4`}};
Th4bMAT = {{4.2`, 15.7`}, {4.`, 15.7`}, {3.9`, 15.7`}, {3.8`, 15.7`},
    {3.6`, 15.8`}, {3.5`, 15.8`}, {3.4`, 15.8`}, {3.2`, 15.9`},
    {3.1`, 16.`}, {3.`, 16.`}, {2.8`, 16.1`}, {2.7`, 16.3`}, {2.6`, 16.5`},
    {2.4`, 16.8`}, {2.3`, 17.2`}, {2.2`, 17.6`}, {2.`, 18.`}, {1.9`, 18.5`},
    {1.8`, 19.`}, {1.6`, 19.6`}, {1.5`, 20.4`}, {1.4`, 21.4`}, {1.2`, 23.`},
    {1.1`, 25.9`}, {1.`, 31.3`}, {0.8`, 35.`}, {0.7`, 34.8`}, {0.6`, 34.4`},
    {0.4`, 34.2`}, {0.3`, 33.9`}, {0.2`, 33.6`}, {0.`, 33.4`}};
Th5bMAT = {{4.2`, 14.1`}, {4.`, 14.2`}, {3.9`, 14.2`}, {3.8`, 14.2`},
    {3.6`, 14.2`}, {3.5`, 14.1`}, {3.4`, 14.1`}, {3.2`, 14.`}, {3.1`, 14.1`},
    {3.`, 14.1`}, {2.8`, 14.2`}, {2.7`, 14.4`}, {2.6`, 14.6`}, {2.4`, 14.7`},
    {2.3`, 15.`}, {2.2`, 15.2`}, {2.`, 15.6`}, {1.9`, 16.`}, {1.8`, 16.4`},
    {1.6`, 16.8`}, {1.5`, 17.5`}, {1.4`, 18.4`}, {1.2`, 19.5`},
    {1.1`, 21.8`}, {1.`, 26.6`}, {0.8`, 33.8`}, {0.7`, 33.5`}, {0.6`, 33.2`},
    {0.4`, 33.`}, {0.3`, 32.7`}, {0.2`, 32.5`}, {0.`, 32.2`}};
Th6bMAT = {{4.2`, 13.4`}, {4.`, 13.4`}, {3.9`, 13.4`}, {3.8`, 13.4`},
    {3.6`, 13.4`}, {3.5`, 13.4`}, {3.4`, 13.4`}, {3.2`, 13.4`},
    {3.1`, 13.3`}, {3.`, 13.3`}, {2.8`, 13.3`}, {2.7`, 13.4`}, {2.6`, 13.4`},
    {2.4`, 13.5`}, {2.3`, 13.8`}, {2.2`, 14.1`}, {2.`, 14.2`}, {1.9`, 14.5`},
    {1.8`, 14.8`}, {1.6`, 15.1`}, {1.5`, 15.5`}, {1.4`, 16.`}, {1.2`, 16.5`},
    {1.1`, 17.7`}, {1.`, 20.8`}, {0.8`, 32.2`}, {0.7`, 31.9`}, {0.6`, 31.7`},
    {0.4`, 31.4`}, {0.3`, 31.2`}, {0.2`, 30.9`}, {0.`, 30.7`}};
```

Plot Experimental Data

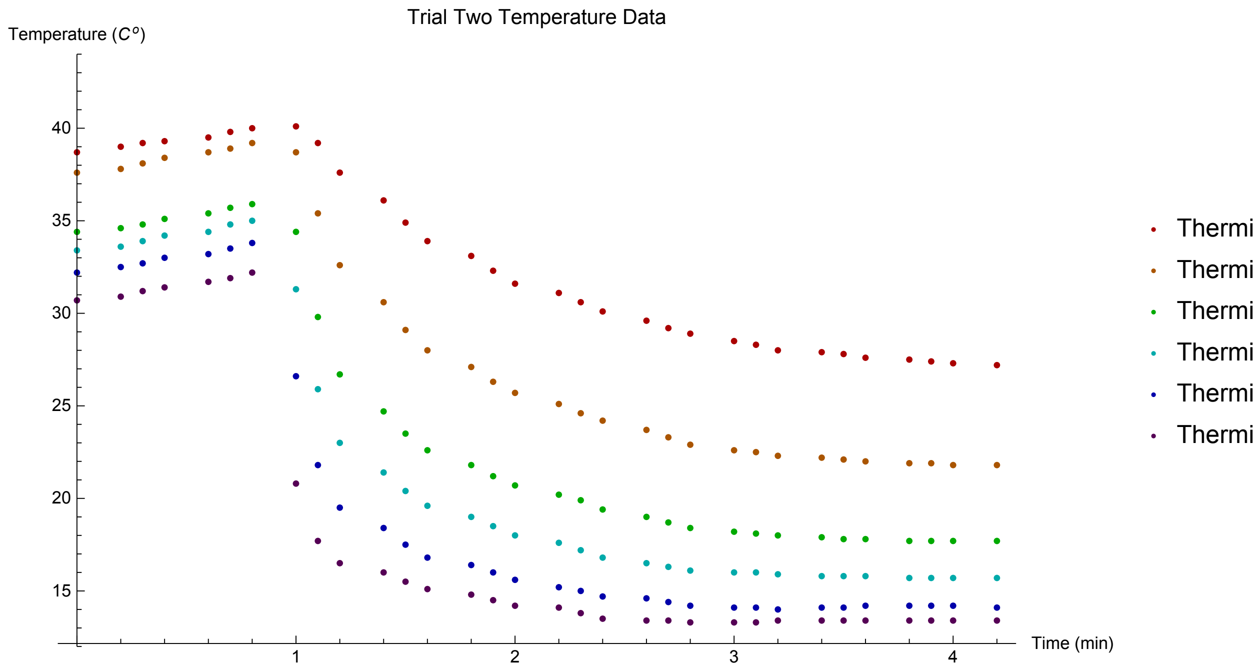
In[374]:=

```
(* Set Temperature Display Range *)
PRange = {12, 44};


(* Load Trial One *)
FirstTrial = ListPlot[{Th1aMAT, Th2aMAT, Th3aMAT, Th4aMAT, Th5aMAT, Th6aMAT},
  PlotRange → PRange, ImageSize → Large, PlotStyle →
   {{Darker[Red], PointSize[0.0067]}, {Darker[Orange], PointSize[0.0067]},
    {Darker[Green], PointSize[0.0067]}, {Darker[Cyan], PointSize[0.0067]},
    {Darker[Blue], PointSize[0.0067]}, {Darker[Purple], PointSize[0.0067]}} ,
  PlotLegends → {"Thermistor 1", "Thermistor 2", "Thermistor 3", "Thermistor 4",
    "Thermistor 5", "Thermistor 6"}, PlotLabel → "Trial One Temperature Data",
  AxesLabel → {"Time (min)", "Temperature (C°)"}]



(* Load Trial Two *)
SecondTrial = ListPlot[{Th1bMAT, Th2bMAT, Th3bMAT, Th4bMAT, Th5bMAT, Th6bMAT},
  PlotRange → PRange, ImageSize → Large, PlotStyle →
   {{Darker[Red], PointSize[0.0067]}, {Darker[Orange], PointSize[0.0067]},
    {Darker[Green], PointSize[0.0067]}, {Darker[Cyan], PointSize[0.0067]},
    {Darker[Blue], PointSize[0.0067]}, {Darker[Purple], PointSize[0.0067]}},
  PlotLegends → {"Thermistor 1", "Thermistor 2", "Thermistor 3", "Thermistor 4",
    "Thermistor 5", "Thermistor 6"}, PlotLabel → "Trial Two Temperature Data",
  AxesLabel → {"Time (min)", "Temperature (C°)"}]
```

Out[375]=



Out[376]=

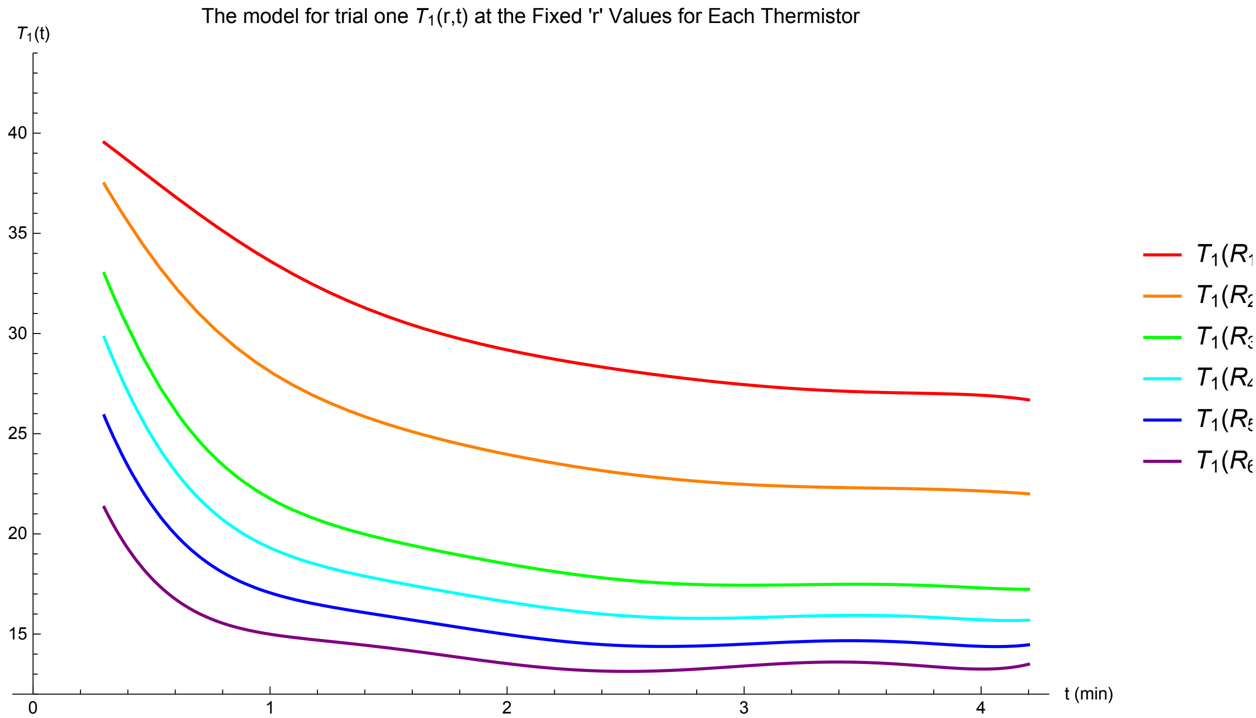


Plot Model Results

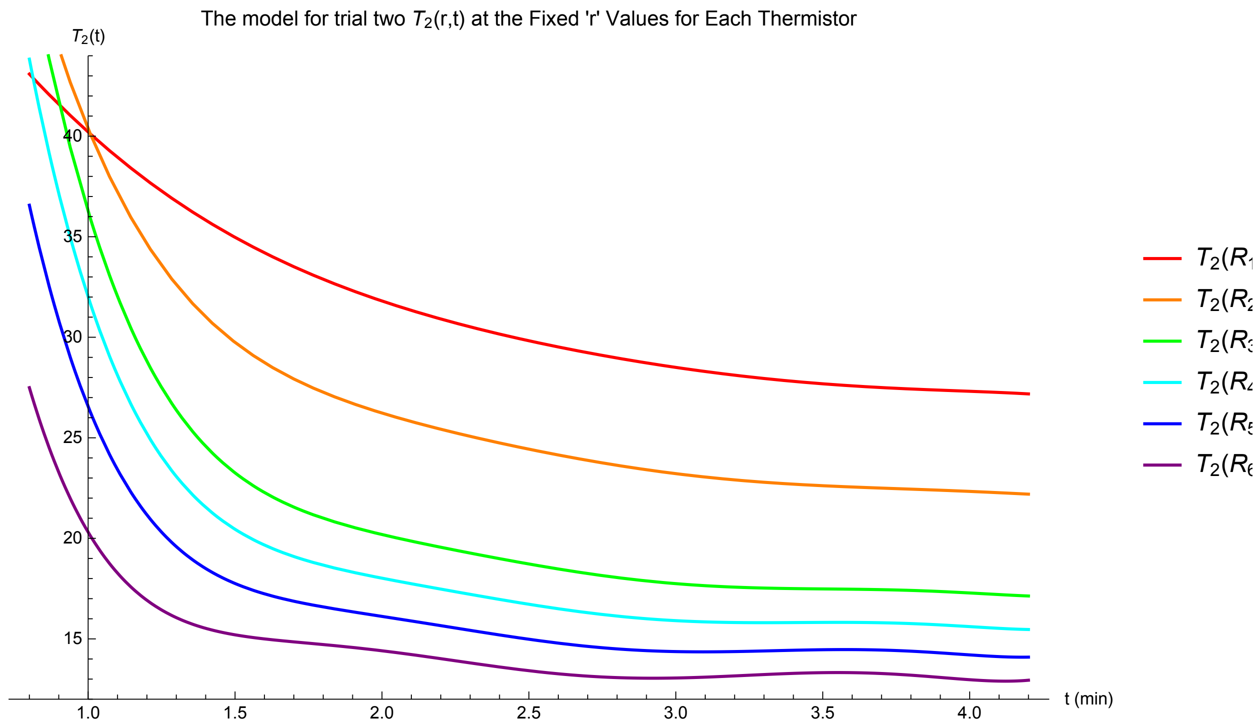
In[322]:=

```
(* Plot Trial One Model *)
Awesome1 =
 Plot[{T1[R1, t], T1[R2, t], T1[R3, t], T1[R4, t], T1[R5, t], T1[R6, t]}, {t, 0.3, 4.2},
  PlotRange → PRange, PlotStyle → {Red, Orange, Green, Cyan, Blue, Purple},
  PlotLegends → "Expressions", PlotLabel →
   "The model for trial one T1(r,t) at the Fixed 'r' Values for Each Thermistor",
  AxesLabel → {"t (min)", "T1(t)"}, ImageSize → Large]


(* Plot Trial Two Model *)
Awesome2 =
 Plot[{T2[R1, t], T2[R2, t], T2[R3, t], T2[R4, t], T2[R5, t], T2[R6, t]}, {t, 0.8, 4.2},
  PlotRange → PRange, PlotStyle → {Red, Orange, Green, Cyan, Blue, Purple},
  PlotLegends → "Expressions", PlotLabel →
   "The model for trial two T2(r,t) at the Fixed 'r' Values for Each Thermistor",
  AxesLabel → {"t (min)", "T2(t)"}, ImageSize → Large]
```

Out[322]=



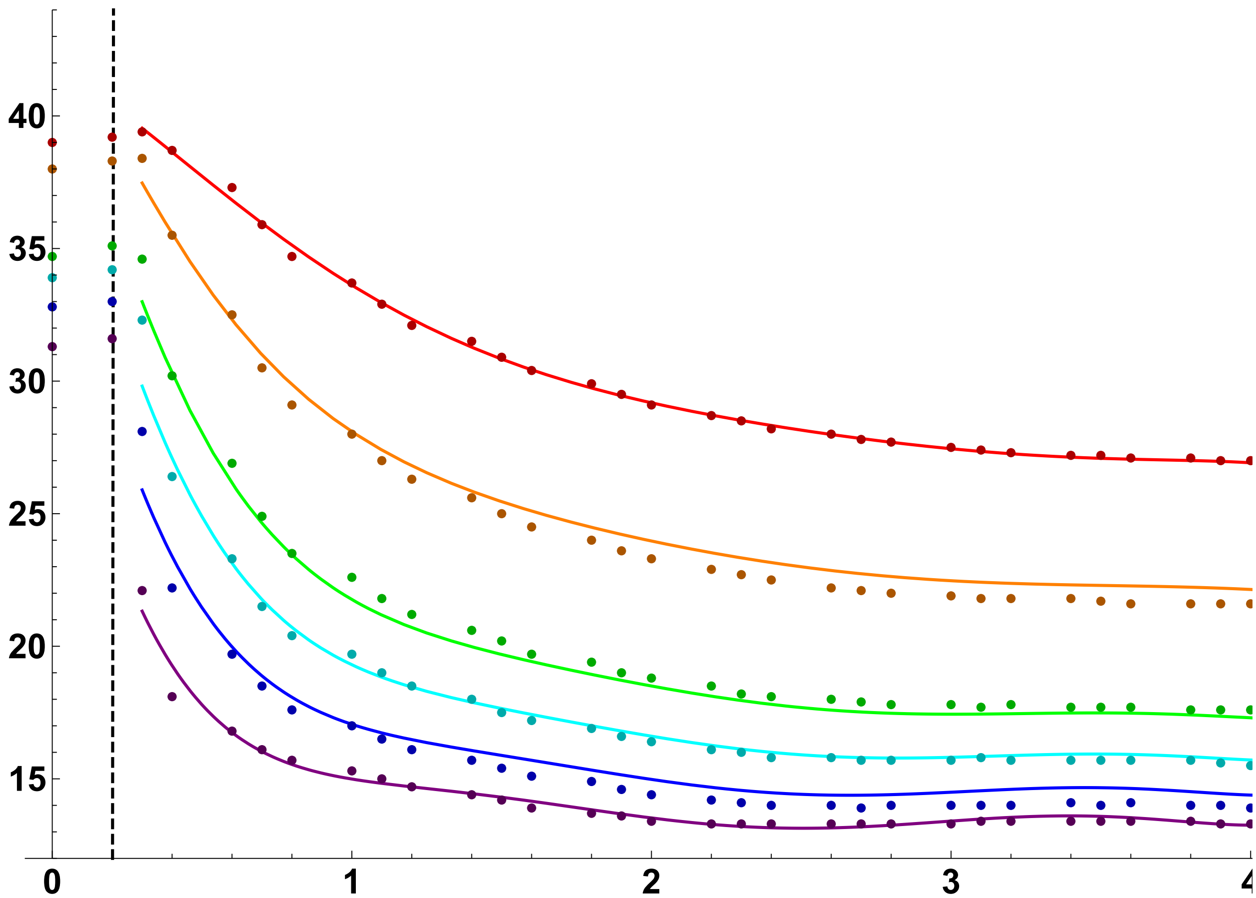
Out[323]=



In[427]:=

```
StartBorder1 =
  Plot[10 000 (t - 0.2), {t, 0, 4.4}, PlotRange → PRange, PlotStyle → {Black, Dashed}];
StartBorder2 = Plot[10 000 (t - 0.8), {t, 0, 4.4},
   PlotRange → PRange, PlotStyle → {Black, Dashed}];

Show[StartBorder1, Awesome1, FirstTrial, LabelStyle → {Directive[Bold, Black], 18}]
Show[StartBorder2, Awesome2,
  SecondTrial, LabelStyle → {Directive[Bold, Black], 18}];
```

Out[429]=




```
(* ERROR METRIC *)
```

In[465]:=

$$\text{ErrPolynomial} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th1aMAT[[n, 2]]} - T_1[R_1, \text{Th1aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th1aMAT[[n, 2]]})^2}}\, 100$$

$$\text{ErrT2} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th2aMAT[[n, 2]]} - T_1[R_2, \text{Th2aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th2aMAT[[n, 2]]})^2}}\, 100$$

$$\text{ErrT3} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th3aMAT[[n, 2]]} - T_1[R_3, \text{Th3aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th3aMAT[[n, 2]]})^2}}\, 100$$

$$\text{ErrT4} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th4aMAT[[n, 2]]} - T_1[R_4, \text{Th4aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th4aMAT[[n, 2]]})^2}}\, 100$$

$$\text{ErrT5} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th5aMAT[[n, 2]]} - T_1[R_5, \text{Th5aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th5aMAT[[n, 2]]})^2}}\, 100$$

$$\text{ErrT6} = \frac{\sqrt{\sum_{n=4}^{32} (\text{Th6aMAT[[n, 2]]} - T_1[R_6, \text{Th6aMAT[[n, 1]]}])^2}}{\sqrt{\sum_{n=4}^{32} (\text{Th6aMAT[[n, 2]]})^2}}\, 100$$

```
Th1aMAT[[3, 1]];
Th1aMAT;
```

Out[465]= 1.90742

Out[466]= 4.72008

Out[467]= 6.32834

Out[468]= 6.26158

Out[469]= 7.9147

Out[470]= 8.84455

■ END